ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

Univariate Self-Starting Shiryaev (U3S): A Bayesian Online Change Point Model in Short Runs

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A brief description

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- individual univariate short horizon data
- detecting persistent shifts without phase I calibration (self-starting)

Our proposal:

- is based on Shiryaev's process (1963)
- relaxes the strict assumption of known parameters
- detects a potential change point (At Most One Change - AMOC scenario) and provides posterior inference for all parameters of interest.

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The likelihood will be:

$$f\left(\boldsymbol{x_n}|\boldsymbol{\theta},\boldsymbol{\phi},\tau\right) = \begin{cases} f\left(\boldsymbol{x_n}|\boldsymbol{\theta},\boldsymbol{\phi},\tau \le n\right) = \prod_{i=1}^{\tau-1} f\left(x_i|\boldsymbol{\theta}\right) \prod_{i=\tau}^{n} f\left(x_i|g(\boldsymbol{\theta},\boldsymbol{\phi})\right) & \text{if } \tau \le n \\ f\left(\boldsymbol{x_n}|\boldsymbol{\theta},\tau > n\right) = \prod_{i=1}^{n} f\left(x_i|\boldsymbol{\theta}\right) & \text{if } \tau > n \end{cases}$$

The stopping time is based on the posterior marginal probability of a change point occurrence, which is:

$$
\begin{aligned}
p\left(\tau \leq n | \boldsymbol{x_n}\right) &= \frac{f\left(\boldsymbol{x_n} | \tau \leq n\right)\pi(\tau \leq n)}{f\left(\boldsymbol{x_n} | \tau \leq n\right)\pi(\tau \leq n) + f\left(\boldsymbol{x_n} | \tau > n\right)\pi(\tau > n)} \\
&= \frac{\displaystyle\sum_{k=1}^{n} \frac{\pi(\tau = k)}{\pi(\tau > n)} \cdot BF_{k,n+}}{\displaystyle\sum_{k=1}^{n} \frac{\pi(\tau = k)}{\pi(\tau > n)} \cdot BF_{k,n+} + 1}
\end{aligned}
$$

where $BF_{k,n+} = \dfrac{f(\boldsymbol{x_n} | \tau = k)}{f(\boldsymbol{x_n} | \tau > n)}$ (Bayes Factor), compares the evidence that the $k^{th} \leq n$ observation to be the change point against the evidence that all available n observations are IC.

- The marginal distributions involved in the computation are:

$$f(\boldsymbol{x_n}|\tau > n) = \int_{\Theta} f(\boldsymbol{x_n}|\boldsymbol{\theta}, \tau > n)\pi(\boldsymbol{\theta})d\boldsymbol{\theta}$$

$$f(\boldsymbol{x_n}|\tau \leq n) = \int_{\Phi}\int_{\Theta} f(\boldsymbol{x_n}|\boldsymbol{\theta}, \boldsymbol{\phi}, \tau \leq n)\pi(\boldsymbol{\theta})\pi(\boldsymbol{\phi})d\boldsymbol{\theta}d\boldsymbol{\phi}$$

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- If the prior $\pi(\boldsymbol{\theta})$ is improper, we sacrifice the s first observations $\boldsymbol{x_{1:s}}$ needed to make the posterior $p(\boldsymbol{\theta}|\boldsymbol{x_{1:s}})$ proper and then replace the prior $\pi(\boldsymbol{\theta})$ by $p(\boldsymbol{\theta}|\boldsymbol{x_{1:s}})$.

Self-starting Shiryaev (3S)

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Constant decision limit p^*

$$T(p^*) = inf\left\{n \geq 1 : p\left(\tau \leq n | \boldsymbol{x_n}\right) \geq p^*\right\}$$

Adapted decision limit p_n^*

$$T(p_n^*) = inf\left\{ n \geq 1 : p\left(\tau \leq n | \boldsymbol{x_n}\right) \geq p_n^* = \frac{K \cdot \sum_{k=1}^{n} \frac{\pi(\tau = k)}{\pi(\tau > n)}}{K \cdot \sum_{k=1}^{n} \frac{\pi(\tau = k)}{\pi(\tau > n)} + 1} \right\}$$

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- Apart from change point detection, we can also provide inference for the unknown parameters:

  - $$\begin{cases} p_{IC}\left(\boldsymbol{\theta} | \boldsymbol{x_n}\right) & \text{if a change point did not occur} \\ p_{OOC}\left(\boldsymbol{\theta}, \boldsymbol{\phi}, \tau | \boldsymbol{x_n}\right) & \text{if an alarm is raised} \end{cases}$$

Self-starting Shiryaev (3S)

IC scenario ($\tau > n$)

OOC scenario ($\tau \leq n$)

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↓

$f\left(\boldsymbol{x_n} | \boldsymbol{\theta}, \tau > n\right)$

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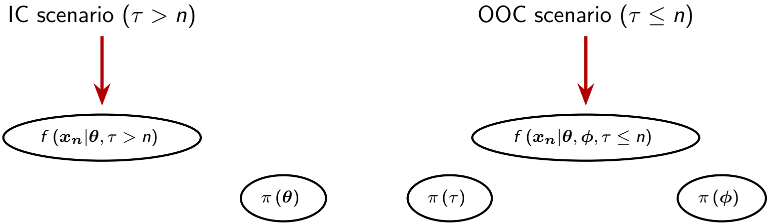

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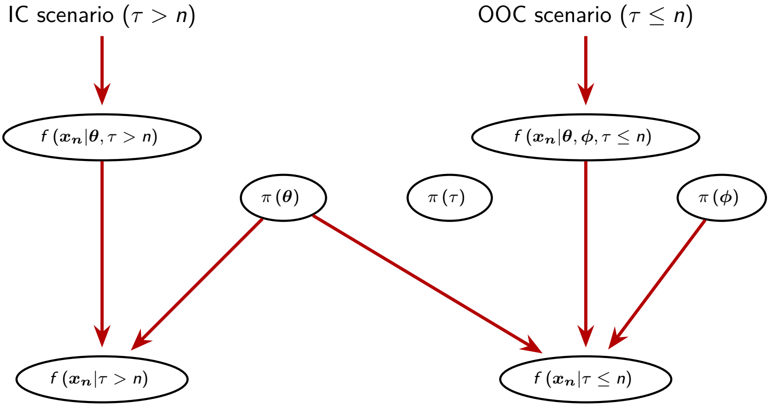

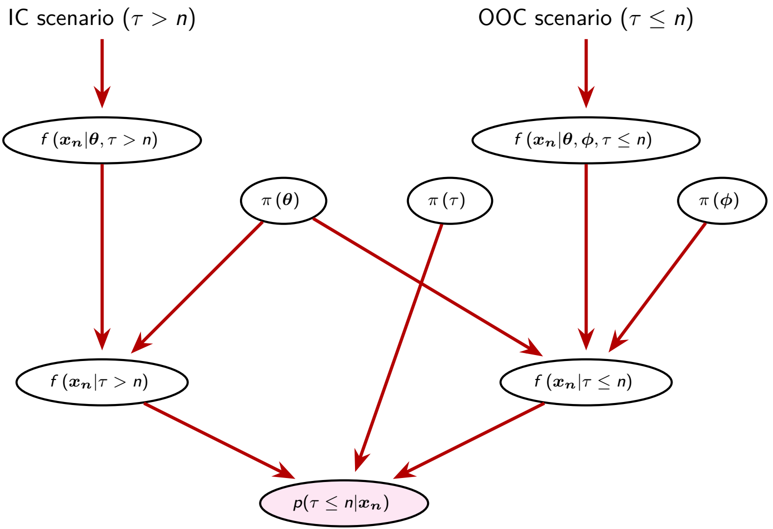
IC scenario $(\tau > n)$
OOC scenario $(\tau \leq n)$
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$\pi(\boldsymbol{\theta})$
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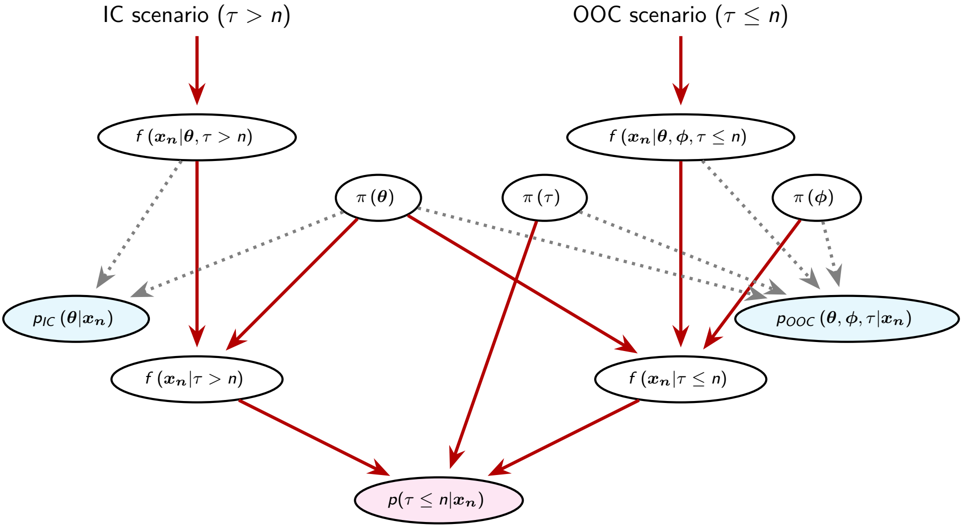
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- **Model parameters**

U3S for the Normal mean

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$\boldsymbol{\theta} = \left(\theta_1, \theta_2^2\right)$: the mean and the variance of the data

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- **Model states**

  IC state: $x_i|\boldsymbol{\theta} \overset{iid}{\sim} N\left(\theta_1, \theta_2^2\right)$

  OOC state: $x_i|(\boldsymbol{\theta}, \phi) \overset{iid}{\sim} N\left(\theta_1 + \delta \cdot \theta_2, \theta_2^2\right)$

Prior setting

- $\pi(\boldsymbol{\theta}) \propto L(\boldsymbol{\theta}|\boldsymbol{Y})^{\alpha_0}\,\pi_0(\boldsymbol{\theta})$ (power prior, Ibrahim 2000), where:
  $\boldsymbol{Y} = (y_1, ..., y_{n_0})$ is the vector of the historical data (if available),
  $0 \leq \alpha_0 \leq 1$ is fixed and controls the influence of the historical data,
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- $\delta = \gamma \cdot \delta_1 + (1-\gamma) \cdot \delta_2$ (mixture of shifts), where:
  $\delta_i \sim N\left(\mu_{\delta i}, {\sigma_{\delta i}}^2\right)$,
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  π is the prior probability of the shift δ_1 in the mixture.
- $\tau \sim DW(p, \beta)$ (*Discrete Weibull*), where
  τ is the location of a potential change point,
  p is the probability for an observation to be OOC,
  β controls the hazard function,
  if $\beta = 1$ then $\tau \sim G(p)$ (*Geometric*)

Under the **IC scenario** the posterior distribution is:

- $(\theta_1, \theta_2^2)|(\tau > n, \boldsymbol{x_n}) \sim NIG\left(\frac{\lambda\mu_0 + X_{1:n}}{\lambda + n}, \lambda + n, a_p, b_p\right)$

where $X_{t_1:t_2} = \sum_{i=t_1}^{t_2} x_i$, $a_p = a + \frac{n}{2}$ and $b_p = b + \frac{1}{2}\left(\sum_{i=1}^{n}(x_i - \bar{x})^2 + \frac{\lambda n}{\lambda + n}(\bar{x} - \mu_0)^2\right)$

Posterior distributions

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Under the **OOC scenario** the posterior distributions are:

- $\theta_1|(\theta_2^2, \delta, \tau \leq n, \boldsymbol{x_n}) \sim N\left(\frac{\lambda\mu_0 + X_{1:n} - n_\tau\delta\theta_2}{\lambda + n}, \frac{\theta_2^2}{\lambda + n}\right)$
- $p\left(\theta_2^2|\theta_1, \delta, \tau \leq n, \boldsymbol{x}\right) \propto \left(\frac{1}{\theta_2^2}\right)^{a_p + \frac{3}{2}} exp\left\{-\frac{2b + S_{1:n}^2 + \lambda\left(\theta_1 - \mu_0\right)^2}{2\theta_2^2} - \frac{\left(X_{\tau:n} - n_\tau\theta_1\right)\delta}{\theta_2}\right\}$

where $n_t = n - t + 1$ and $S_{t_1:t_2}^2 = \sum_{i=t_1}^{t_2}(x_i - \theta_1)^2$

Posterior distributions

- $\delta_i | \left(\theta_1, \theta_2^2, \tau \leq n, \boldsymbol{x_n}\right) \sim N\left(\mu_{\delta pi}, \sigma_{\delta pi}^2\right)$

- $\gamma | \left(\theta_1, \theta_2^2, \delta_i, \tau \leq n, \boldsymbol{x_n}\right) \sim Ber\left(\dfrac{\pi}{\pi + (1-\pi) \cdot exp\left\{\dfrac{\mu_{\delta p2}^2}{2\sigma_{\delta p2}^2} - \dfrac{\mu_{\delta p1}^2}{2\sigma_{\delta p1}^2}\right\} \dfrac{\sigma_{\delta p2}}{\sigma_{\delta p1}}}\right)$

- $p\left(\tau = k | \theta_1, \theta_2^2, \delta, \boldsymbol{x_n}\right) = \dfrac{exp\left\{\dfrac{\delta\left(X_{k:n} - n_k\theta_1\right)}{\theta_2} - \dfrac{n_k\delta^2}{2}\right\}\left((1-p)^{(k-1)^\beta} - (1-p)^{k^\beta}\right)}{\sum_{j=1}^{n} exp\left\{\dfrac{\delta\left(X_{j:n} - n_j\theta_1\right)}{\theta_2} - \dfrac{n_j\delta^2}{2}\right\}\left((1-p)^{(j-1)^\beta} - (1-p)^{j^\beta}\right)}$

where $\mu_{\delta pi} = \dfrac{\mu_{\delta i} + \sigma_{\delta i}^2\left(X_{\tau:n} - n_\tau\theta_1\right)/\theta_2}{1 + n_\tau\sigma_{\delta i}^2}$ and $\sigma_{\delta p1}^2 = \dfrac{\sigma_{\delta i}^2}{1 + n_\tau\sigma_{\delta i}^2}$

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- The observations arrive sequentially, assuming:

$$\boldsymbol{X}_i|\boldsymbol{\theta} \overset{iid}{\sim} N\left(\theta_1, \theta_2^2\right)$$

Non-informative prior setting:

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- $\pi(\boldsymbol{\theta}) \propto 1/\theta_2^2 \equiv NIG(0, 0, -1/2, 0)$ (reference prior, Bernardo, 1979)
- $\delta|\gamma \sim \gamma \cdot N(1, 0.25^2) + (1-\gamma) \cdot N(-1, 0.25^2)$
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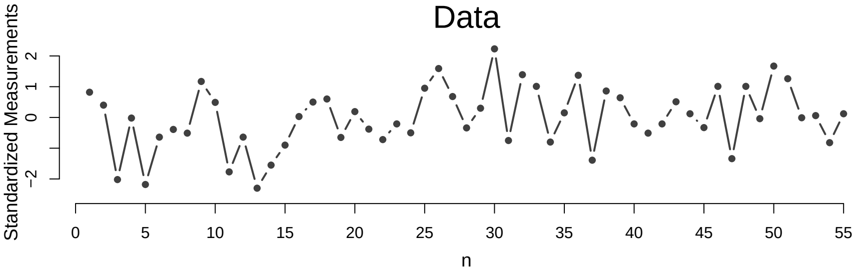
Decision limit elicitation:

- We set ${p_n}^*$ to control $PFA = 20\%$ for $n = 55$ data points.

Application to real data



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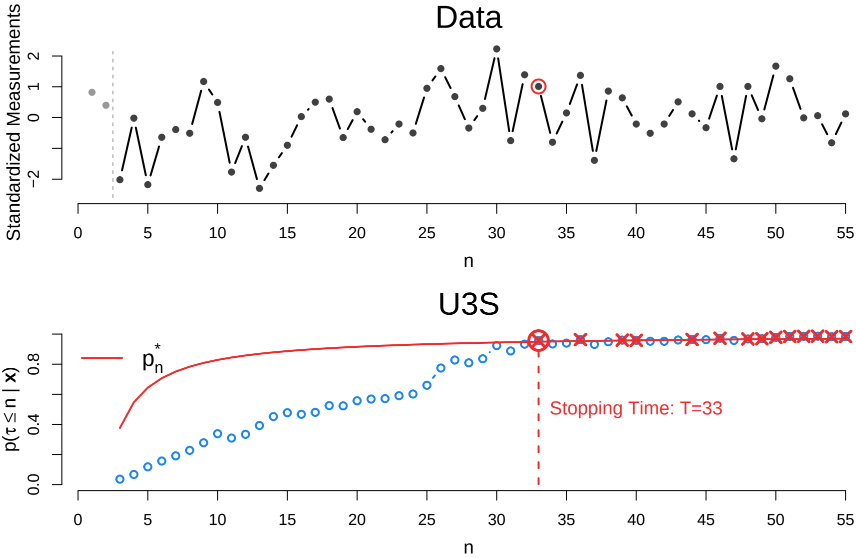

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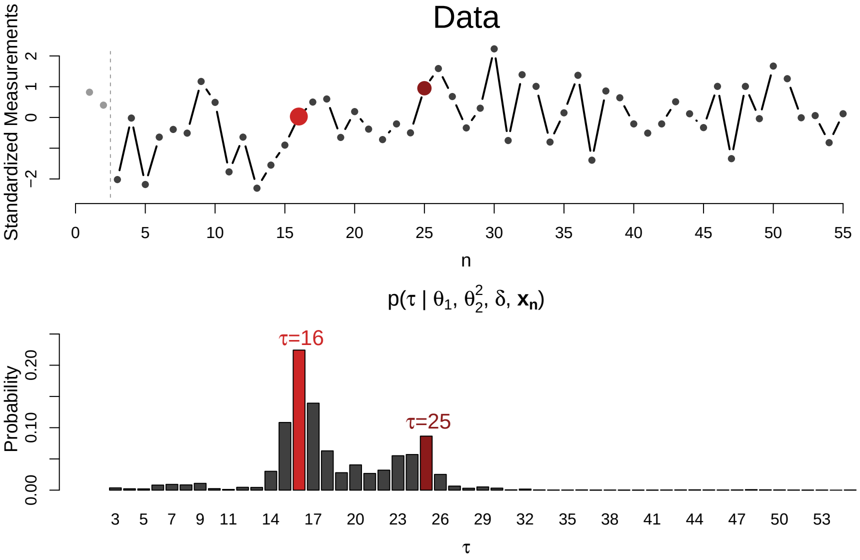

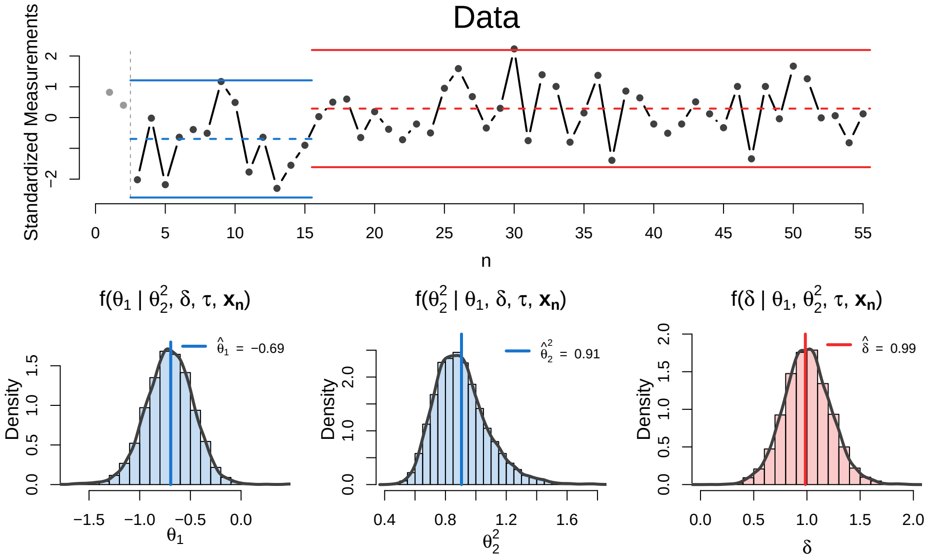

Data
Standardized Measurements
−2
0
1
2
0
5
10
15
20
25
30
35
40
45
50
55
n
$f(\theta_1 \mid \theta_2^2, \delta, \tau, \mathbf{x_n})$
$\hat{\theta}_1 = -0.69$
Density
0.0
0.5
1.0
1.5
−1.5
−1.0
−0.5
0.0
θ_1
$f(\theta_2^2 \mid \theta_1, \delta, \tau, \mathbf{x_n})$
$\hat{\theta}_2^2 = 0.91$
Density
0.0
1.0
2.0
0.4
0.8
1.2
1.6
θ_2^2
$f(\delta \mid \theta_1, \theta_2^2, \tau, \mathbf{x_n})$
$\hat{\delta} = 0.99$
Density
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0
1.5
2.0
δ

Competing methods:

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IC data:

- For $N = 50$, we assume $X_i | \left(\theta_1, \theta_2^2\right) \overset{i.i.d.}{\sim} N\left(\theta_1, \theta_2^2\right)$, where $\theta_1 = 0$ and $\theta_2^2 = 1$. We simulate $10,000$ iterations of each random sample.

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OOC scenarios:

- Step changes for the mean from a $N(1, 1)$ and initiating at location 11, or 26, or 41.

U3S prior setting (reference & constant hazard function (r,c)):

- $\pi(\boldsymbol{\theta}) \propto 1/\theta_2^2$
- $\delta|\gamma \sim \gamma \cdot N(1, 0.25^2) + (1-\gamma) \cdot N(-1, 0.25^2)$
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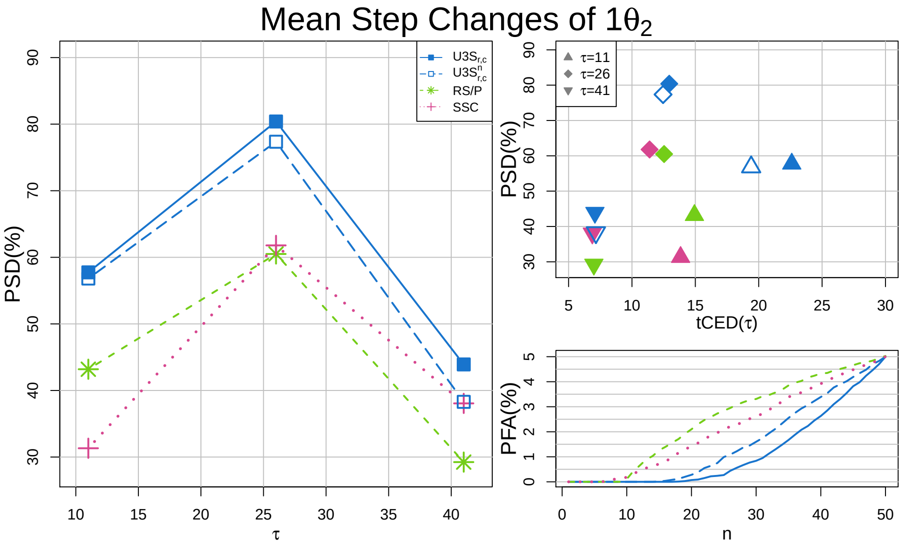
$$tCED(\tau) = E_{\tau}(T - \tau + 1|\tau \leq T \leq n)$$


Mean Step Changes of 1θ2
U3S r,c
U3S n r,c
RS/P
SSC
PSD(%)
τ
τ=11
τ=26
τ=41
tCED(τ)
PFA(%)
n

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Thank you!