Statistical Inference for a Wiener-based degradation model with imperfect maintenance actions under different observation schemes

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Degradable systems

The Wiener-based ARD$_1$ model *(Arithmetic Reduction of the Degradation)*

- Wiener process with drift

  Let $X(t)$ be the degradation level at time t.

  $$X(t) = \mu t + \sigma B(t) \Longrightarrow X(t+\Delta t) - X(t) \sim \mathcal{N}(\mu \Delta t, \sigma^2 \Delta t)$$

  The increments of degradation are independent on disjoints time intervals

- Imperfect maintenance actions

  The maintenance effect is to reduce the degradation level of a certain quantity that is proportional to the degradation level accumulated since the last maintenance action (Mercier and Castro, 2019).
  Let $(\tau_1, ..., \tau_j, ..., \tau_k)$ be the maintenance times and ρ the maintenance efficiency parameter so that $\rho \in [0, 1]$

  $\forall t \in [\tau_j, \tau_{j+1}[$ and $\forall j \in \{1, ..., k\}$, we have :

  $$Y(\tau_j^+) - Y(\tau_j^-) \ = \ Z_j^1 \ = \ -\rho \ \left(Y(\tau_j^-) - Y(\tau_{j-1}^+)\right) \quad \text{and } Y(t) = X(t) - \rho \, X(\tau_j)$$

  where $Y(\tau_j^+)$ and $Y(\tau_j^-)$ are the degradation levels just after and just before the j^{th} maintenance, and Z_j^1 the jump observed at τ_j.

Background

- Few papers on degradation models with maintenance effects (Zhang et al., 2015; Giorgio and Pulcini, 2018; Salles et al., 2020; Kamranfar et al., 2021)
- Different statistical inference methods (Kahle and Lehmann, 2010; Zhang et al., 2015; Salles et al., 2020)
- Influence of the observation scheme on the inference's quality (Zhang et al., 2015; Zhao et al., 2019; Salles et al., 2020)

Simulated degradation levels with different values of the maintenance efficiency paramater

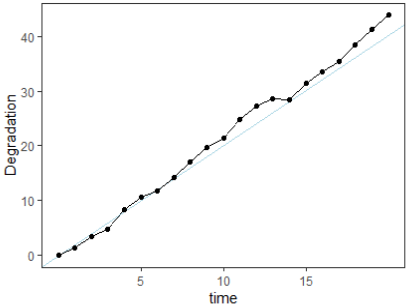


Figure 1 – Minimal repair (ABAO) , $\rho = 0$

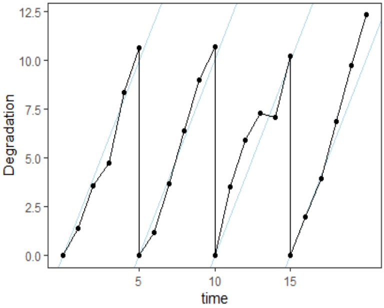


Figure 2 – Perfect repair (AGAN) , $\rho = 1$

Notations

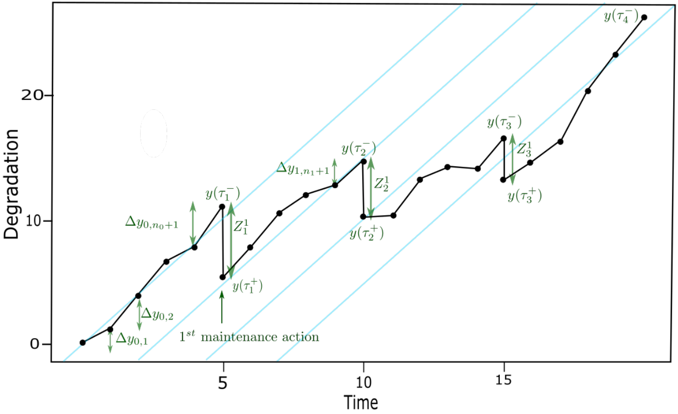
$\forall i \in \{0, ..., n_j + 1\}, \quad \forall j \in \{0, ...k\}$:

$\Delta Y_{j,i}$: The i^{th} increment of degradation since the j^{th} maintenance action

n_j : The number of observations between two successive maintenance actions, i.e. on $]\tau_j, \tau_{j+1}[$

$t_{j,i}$: The i^{th} observation time after the j^{th} maintenance action

$\mathcal{H}_{\tau_j}$: All the increments observed before τ_j



Parameters estimations

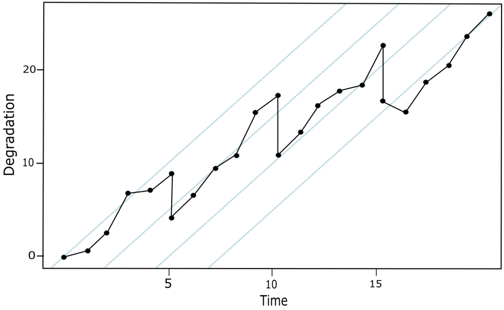
- The estimators $\hat{\mu}$, $\hat{\sigma}^2$ and $\hat{\rho}$ are computed with the maximum likelihood estimation
- From now and then, four observation schemes will be distinguished, each one leads to a different writing of the likelihood and therefore to a different estimation of the parameters
- The likelihood $L(\mu, \sigma^2, \rho)$ is written as the product of the increments densities multiplied by the joint density of the jumps conditionally to the previous observations

$$L(\mu, \sigma^2, \rho) = \prod_j \prod_i f_{\Delta Y_{j,i}}(\Delta y_{j,i}) \prod_j f_{Z_j^m | \mathcal{H}^m_{\tau_j^-}}(z_j^m)$$

Where m is the the observation scheme, $m \in \{1, 2, 3, 4\}$

Observation scheme n°1 : the degradation levels are observed just before and just after each maintenance action

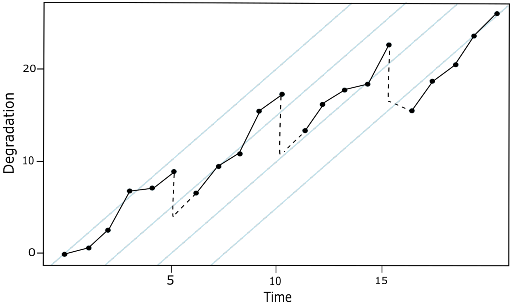
$$L_1\left(\mu, \sigma^2, \rho\right) = \prod_{j=0}^{k} \prod_{i=1}^{n_j+1} f_{\Delta Y_{j,i}}(\Delta y_{j,i}) \prod_{j=1}^{k} \delta\left(z_j^1 = -\rho \sum_{i=1}^{n_{j-1}+1} \Delta y_{j-1,i}\right)$$



Observation scheme n°2 : the degradation level just after each maintenance action is not observed

$$L_2\left(\mu, \sigma^2, \rho\right) = \left[\prod_{j=0}^{k-1} \prod_{i=1+\mathbb{1}_{j>0}}^{n_j+1} f_{\Delta Y_{j,i}}(\Delta y_{j,i})\right] \times \left[\prod_{j=1}^{k-1} f_{Z_j^2 \mid \mathcal{H}_{\tau_j^-}}\left(z_j^2\right)\right]$$

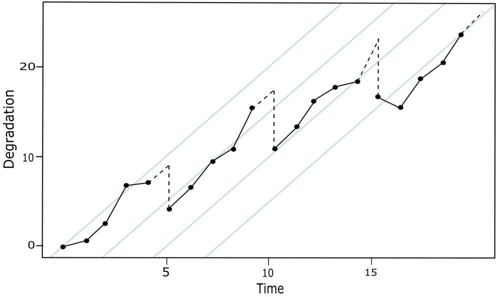
$$Z_j^2 \mid \mathcal{H}_{\tau_j^-} \sim \mathcal{N}\left(\mu\ \Delta t_{j,1} - \rho\ y(\tau_j^-) + (1-\rho) \sum_{i=1}^{j-1} \rho^{j-i}\ y(\tau_i^-),\ \sigma^2 \Delta t_{j,1}\right)$$



Observation scheme n°3 : The degradation level just before the maintenance action is not observed

$$L_3\left(\mu, \sigma^2, \rho\right) = \left[\prod_{j=0}^{k} \prod_{i=1}^{n_j} f_{\Delta Y_{j,i}}(\Delta y_{j,i})\right] \left[\prod_{j=1}^{k} f_{Z_j^3 | \mathcal{H}_{t_{j-1,n_{j-1}}}}(z_j^3)\right]$$

$$Z_j^3 \mid \mathcal{H}_{t_{j-1,n_{j-1}}} \sim \mathcal{N}\left(\mu[(1-\rho)\Delta t_{j-1,n_{j-1}+1}] - \rho \sum_{i=1}^{n_{j-1}} \Delta y_{j-1,i}\ ,\ \sigma^2[(1-\rho)^2 \Delta t_{j-1,n_{j-1}+1}]\right)$$



Observation scheme n°4 : the degradation level is not observed nor just before neither just after the maintenance action

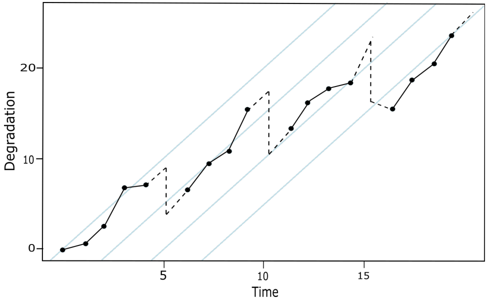
$$L_4\left(\mu,\sigma^2,\rho\right) = g_{(Z_1^4, Z_2^4, \ldots, Z_k^4)}\left(z_1^4, z_2^4, \ldots, z_k^4\right) \times \prod_{j=0}^{k} \prod_{i=1+\mathbb{1}_{j>1}}^{n_j} f_{\Delta Y_{j,i}}\left(\Delta y_{j,i}\right)$$

Where $\forall j \in \{1, \ldots, k\}$, Z_j^4 are the jumps observed between two successive maintenance actions.
$Z^4 = (Z_1^4, Z_2^4, \ldots, Z_k^4)$ is a Gaussian vector and g is the joint density of the jumps conditionally to the previous observed increments

Variance-covariance Matrix Σ :

$$\begin{pmatrix} s_1 & -\rho\Delta t_{1,1} & 0 & \cdots & 0 \\ -\rho\Delta t_{1,1} & s_2 & -\rho\Delta t_{2,1} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \cdots & & 0 \\ 0 & \cdots\ 0 & -\rho\Delta t_{k-3,1} & s_{k-2} & -\rho\Delta t_{k-2,1} \\ 0 & \cdots & 0 & -\rho\Delta t_{k-2,1} & s_{k-1} \end{pmatrix}$$

Where
$s_j = \left[\Delta t_{j,1} + (1-\rho)^2 \Delta t_{j-1,n_{j-1}+1} + \rho^2 \Delta t_{j-1,1}\mathbb{1}_{j>1}\right]$



Parameters estimations according to the observation scheme

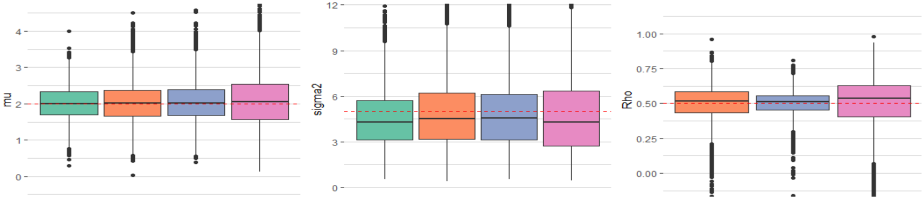


Figure 3 – Estimations including 3 maintenance actions and $\mu = 2$, $\sigma^2 = 5$, $\rho = 0.5$, $\boldsymbol{n_j}$**=2**

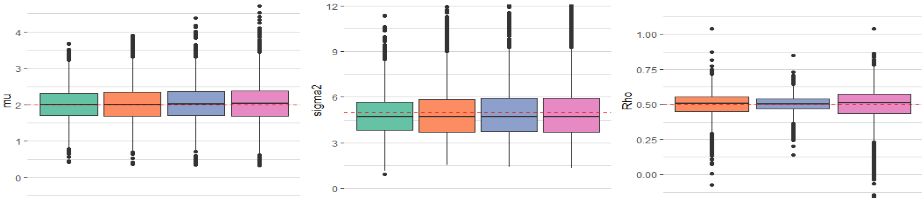


Figure 4 – Estimations including 3 maintenance actions and $\mu = 2$, $\sigma^2 = 5$, $\rho = 0.5$, $\boldsymbol{n_j}$**=5**

Parameters estimations according to the observation scheme

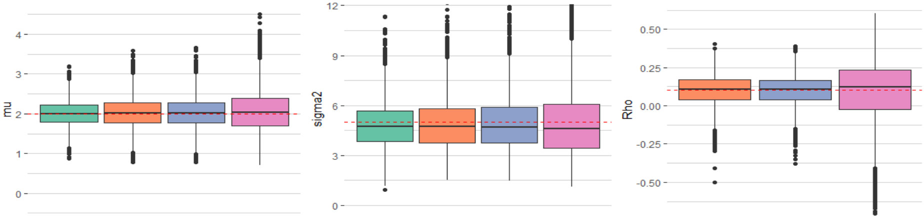


Figure 5 – Estimations including 7 maintenance actions and $\mu = 2$, $\sigma^2 = 5$, $\boldsymbol{\rho}$**=0.1**, $n_j = 2$

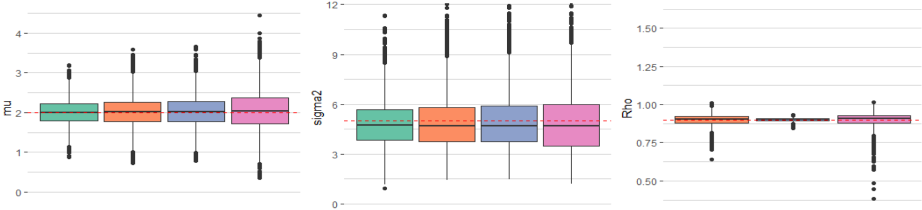


Figure 6 – Estimations including 7 maintenance actions and $\mu = 2$, $\sigma^2 = 5$, $\boldsymbol{\rho}$**=0.9**, $n_j = 2$

Conclusions and future work

- Different estimations according to the observation scheme
- Drawbacks of the Wiener-based ARD_1 model :
  - Deterministic jumps for the first observation scheme
  - Identifiability problem for the third observation scheme
- Other potential model to consider inspired by the ARD_1 model

S. Mercier and I. T. Castro. Stochastic comparisons of imperfect maintenance models for a Gamma deteriorating system. *European Journal of Operational Research*, 273(1) :237–248, 2019. doi : 10.1016/j.ejor.2018.06.020. URL https://hal.archives-ouvertes.fr/hal-01812028.

M. Zhang, O. Gaudoin, and M. Xie. Degradation-based maintenance decision using stochastic filtering for systems under imperfect maintenance. *European Journal of Operational Research*, 245(2) :531–541, 2015. ISSN 0377-2217. URL https://www.sciencedirect.com/science/article/pii/S0377221715001708.

M. Giorgio and G. Pulcini. A new state-dependent degradation process and related model misidentification problems. *European Journal of Operational Research*, 267(3) :1027–1038, 2018. ISSN 0377-2217. URL https://www.sciencedirect.com/science/article/pii/S0377221717311724.

G. Salles, S. Mercier, and L. Bordes. Semiparametric estimate of the efficiency of imperfect maintenance actions for a Gamma deteriorating system. *Journal of Statistical Planning and Inference*, 206 :278–297, May 2020. ISSN 03783758. doi : 10.1016/j.jspi.2019.09.014. URL https://linkinghub.elsevier.com/retrieve/pii/S0378375819300977.

H. Kamranfar, M. Fouladirad, and N. Balakrishnan. Inference for a gradually deteriorating system with imperfect maintenance. *Communications in Statistics - Simulation and Computation*, pages 1–19, December 2021. ISSN 0361-0918. doi : 10.1080/03610918.2021.2001528. URL https://doi.org/10.1080/03610918.2021.2001528.

W. Kahle and A. Lehmann. The Wiener Process as a Degradation Model : Modeling and Parameter Estimation. In M.S. Nikulin, N. Limnios, N. Balakrishnan, W. Kahle, and C. Huber, editors, *Advances in Degradation Modeling : Applications to Reliability, Survival Analysis, and Finance*, pages 127–146. Birkhäuser Boston, Boston, MA, 2010. ISBN 978-0-8176-4924-1. URL https://doi.org/10.1007/978-0-8176-4924-1_9.

X. Zhao, O. Gaudoin, L. Doyen, and M. Xie. Optimal inspection and replacement policy based on experimental degradation data with covariates. *IISE Transactions*, 51(3) :322–336, 2019. doi : 10.1080/24725854.2018.1488308. URL https://doi.org/10.1080/24725854.2018.1488308.