Degradation model with Imperfect and worse Maintenances

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Problem statement

- We observe one system over $[0, T]$
- Deteriorating system (Gamma process)
- Periodic inspections (we denote n the number of inspections over $[0, T]$)
- AGAN corrective maintenance
- Preventive maintenance (Imperfect maintenance or worse maintenance)

Worse Maintenance

Definition

Worse maintenance: a maintenance action which makes the system degradation increases, i.e the degradation level just after the maintenance is greater than the degradation level just before the maintenance.

Some possible reasons:

- Repair the wrong part of the system.
- Repair (partially or completely) the faulty part but damage adjacent parts of the system.
- Human errors such as wrong adjustments and further damage done during maintenance.

Deterioration model

The deterioration process is a Gamma process, $\{X_t, t \geq 0\}$, with shape function $a(t) = \alpha t^\beta$ with $\alpha > 0$ and $\beta > 0$ and scale parameter $b > 0$.
The Gamma process $\{X_t, t \geq 0\}$ is a continuous-time stochastic process such that:

- $X_0 = 0$ with probability 1
- $X_t - X_s \sim G(a(t) - a(s), b)$ for all $t > s \geq 0$, with density function $\forall x \in \mathbb{R}^+$:

$$f(x) = \frac{b^{a(t)-a(s)}}{\Gamma(a(t) - a(s))} x^{a(t)-a(s)-1} e^{-bx}$$

- $\{X_t, t \geq 0\}$ has independent non overlapping increments

Maintenance policy

- $\{Y_t, t \geq 0\}$ the stochastic process of the maintained system.
- Inspection times $(t_j)_{j\in\mathbb{N}}$,
- Periodic inspections $t_j = j\tau$, $j \in \mathbb{N}$.

Maintenance decision rule

- if $Y_{t_j^-} \geq M$ then an AGAN CM $Y_{t_j+} = 0$,
- if $L \leq Y_{t_j^-} < M$
  - imperfect PM (ARD_∞) with probability p, $Y_{t_j+} = (1-\rho)Y_{t_j-}$, ($\rho \in [0,1]$),
  - worse PM (AAD_∞) with probability $1-p$, $Y_{t_j+} = (1+\rho_w)Y_{t_j-}$, ($\rho_w > 0$),
- if $Y_{t_j^-} < L$ no action is performed,

where L and M are fixed thresholds ($0 < L < M$).

Context and Aim

- We observe $Y_{i_\tau} = Y_i$, the degradation level of the system just before the i-th maintenance.
- The type of PM (imperfect or worse) is not observed
- Let's denote $U_i = \mathbb{1}_{L \leq Y_{i_\tau} < M}$, the indicator of preventive maintenance.
- Conditionally to $U_i = 1$, let's denote $B_i = 1$ if the PM is imperfect (with probability p) and $B_i = 0$ if the PM is worse (with probability $1 - p$).
- B_i are not observed, independent and Bernoulli distributed, $\mathcal{B}(p)$.
- Statistical inference: EM Algorithm

Notations

- If $0 \leqslant Y_{t_i^-} \leqslant L$, no maintenance action is performed. Consequently, thanks to the stochastic continuity of the gamma process, $\mathbb{P}[Y_{t_i^+} = Y_{t_i^-}] = 0$ and thus $Y_{t_i^+} \approx Y_{t_i^-}$. Hence, in such of a case,

$$Y_{t_{i+1}^-} - Y_{t_i^-} \sim \mathcal{G}(a(t_{i+1}) - a(t_i), b).$$

- If $L \leqslant Y_{t_i^-} \leqslant M$, a PM action is performed:

$$Y_{t_{i+1}^-} - (1-\rho)^{U_i B_i}(1+\rho_w)^{U_i(1-B_i)} Y_{t_i^-} \sim \mathcal{G}(a(t_{i+1}) - a(t_i), b).$$

- If $Y_{t_i^-} \geqslant M$, a perfect CM is performed and thus the system restart as a new one with $Y_{t_i^+} = 0$.

By denoting $\Delta_i^P = \mathbb{I}_{Y_{t_i^-} \geqslant M}$ (perfect maintenance), we have

$$Y_{t_{i+1}^-} - (1-\Delta_i^P)(1-\rho)^{U_i B_i}(1+\rho_w)^{U_i(1-B_i)} Y_{t_i^-} \sim \mathcal{G}(a(t_{i+1}) - a(t_i), b).$$

Complete Likelihood function

- Observations: $(y_i, u_i)_{i=1,\dots,n}$
- b_i are not observed
- $\theta = (\alpha, \beta, b, \rho, \rho_w, p)$

The complete likelihood function L can be written as follows:

$$L(\alpha, \beta, b, \rho, \rho_w, p; data) = \prod_{i=1}^{n} f_{\delta_i, b}\Big(y_{t_i^-} - (1-\Delta^P_{i-1})(1-\rho)^{u_{i-1}b_{i-1}}(1+\rho_w)^{u_{i-1}(1-b_{i-1})} y_{t_{i-1}^-}$$

where $Y_{t_0^-} = \Delta_0^P = u_0 = b_0 = 0$ and $f_{\delta_i, b}$ is the probability density function of the gamma distribution with shape parameter $\delta_i = \alpha(t_i^\beta - t_{i-1}^\beta)$ and scale parameter b.

EM Algorithm

We denote $\mathcal{H}_{i-1} = \{Y_1, \ldots, Y_{i-1}, U_1, \ldots, U_{i-2}, B_1, \ldots, B_{i-2}\}$, the past of the process at time t_{i-1}.

This algorithm is an iterative algorithm and has two steps:

- At step $k+1$, the E-step computes the conditional expectation of the complete log-likelihood
  $$\sum_{i=1}^{n} \mathbb{E}_{B_{i-1}|\mathcal{H}_{i-1}, Y_i, U_{i-1}, \theta^{(k)}}[\log(L(Y_i, U_{i-1}, B_{i-1}|\mathcal{H}_{i-1}, \theta)]$$
- The M-step maximizes this quantity:
  $$\theta^{(k+1)} = \arg\max_{\theta} \sum_{i=1}^{n} \mathbb{E}_{B_{i-1}|Y_i, U_{i-1}, \mathcal{H}_{i-1}, \theta^{(k)}}[\log(L(Y_i, U_{i-1}, B_{i-1}|\mathcal{H}_{i-1}, \theta)]$$

EM Algorithm

As B_{i-1} are Bernoulli distributed, by denoting $\tilde{p}_i = \mathbb{P}(B_{i-1} = 1 | Y_i, U_{i-1}, \mathcal{H}_{i-1}, \theta^{(k)})$, we have

$$\theta^{(k+1)} = \arg\max_{\theta} \Big\{ \sum_{i=1}^{n} \tilde{p}_i \ell(Y_i, U_{i-1}, B_{i-1} = 1 | \mathcal{H}_{i-1}, \theta) + (1 - \tilde{p}_i)\ell(Y_i, U_{i-1}, B_{i-1} = 0 | \mathcal{H}_{i-1}, \theta) \Big\}$$

We can compute $\tilde{p}_i$ as follows

$$\tilde{p}_i = P\left(B_{i-1} = 1 | Y_i, U_{i-1}, \mathcal{H}_{i-1}, \theta^{(k)}\right)$$
$$= \frac{p^{(k)} f_{\delta_i}\left(y_{t_i} - (1 - \Delta^P_{i-1})(1-\rho)^{u_i - 1} y_{t_{i-1}}\right)}{p^{(k)} f_{\delta_i}\left(y_{t_i} - (1 - \Delta^P_{i-1})(1-\rho)^{u_i - 1} y_{t_{i-1}}\right) + (1 - p^{(k)}) f_{\delta_i}\left(y_{t_i} - (1 - \Delta^P_{i-1})(1+\rho_w)^{u_i - 1} y_{t_{i-1}}\right)}$$

Therefore, the conditional expectation
$\mathbb{E}_{B|Y,U,\theta^{(k)}}\left[\sum_{i=1}^{n}\log(L(Y_i, U_{i-1}, B_{i-1}|\mathcal{H}_{i-1},\theta)\right]$ is

$$= \sum_{i=1}^{n} \tilde{p}_i \ell(Y_i, U_{i-1}, B_{i-1} = 1|\mathcal{H}_{i-1},\theta) + (1-\tilde{p}_i)\ell(Y_i, U_{i-1}, B_{i-1} = 0|\mathcal{H}_{i-1},\theta)$$

$$= \log(p)\sum_{i=1}^{n}\tilde{p}_i + \sum_{i=1}^{n}\tilde{p}_i \log\left(f_{\delta_i}(y_{t_i} - (1-\Delta_{i-1}^{P})(1-\rho)^{u_{i-1}} y_{t_{i-1}})\right)$$

$$+ \log(1-p)\sum_{i=1}^{n}(1-\tilde{p}_i) + \sum_{i=1}^{n}(1-\tilde{p}_i)\log\left(f_{\delta_i}(y_{t_i} - (1-\Delta_{i-1}^{P})(1+\rho_w)^{u_{i-1}} y_{t_{i-1}})\right)$$

EM Algorithm

- Initialisation $\theta^{(0)}$
- At step $k+1$:
  - Compute $\tilde{p}_i$
  - Compute $p^{(k+1)} = \dfrac{\sum_{i=1}^{n} u_{i-1}\tilde{p}_i}{\sum_{i=1}^{n} u_{i-1}}$
  - Compute $b^{(k+1)}$

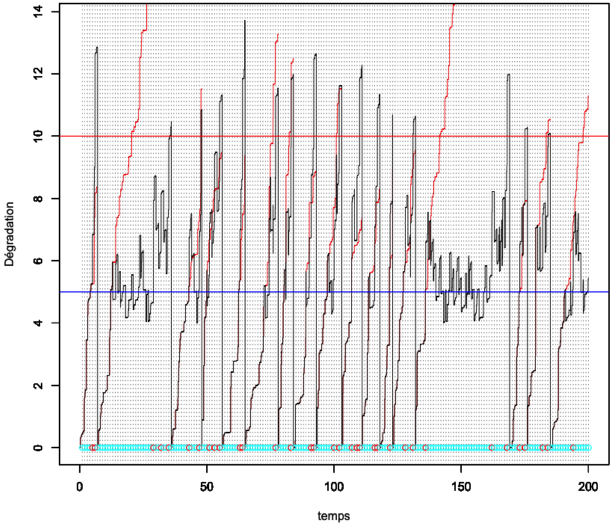
$$\frac{\sum_{i=1}^{n} \delta_i}{\sum_{i=1}^{n} \tilde{p}_i(y_{t_i} - (1-\Delta_{i-1}^{P})(1-\rho)^{u_{i-1}} y_{t_{i-1}}) + (1-\tilde{p}_i)(y_{t_i} - (1-\Delta_{i-1}^{P})(1+\rho_w)^{u_{i-1}} y_{t_{i-1}})}$$

  - Compute $(\rho^{(k+1)}, {\rho_w}^{(k+1)})$ which maximize E
  - Compute $(\alpha^{(k+1)}, \beta^{(k+1)})$ which maximize E

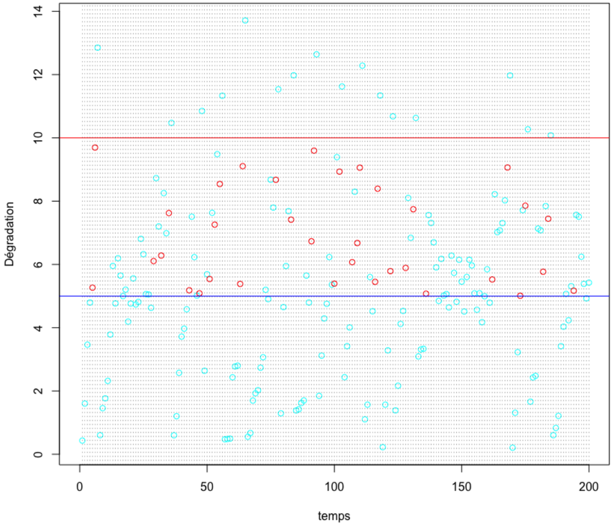
Simulations

- $\tau = 1$ (inter-inspection time)
- $\alpha = 1$, $\beta = 1$ and $b = 1$ (Gamma process)
- $\rho = 0.2$
- $\rho_w = 0.3$
- $p = 0.7$
- Four case studies: $T \in 50, 100, 150, 200$
- $L = 5$ and $M = 10$ (respectively the PM and CM thresholds)
- $\theta^{(0)} = (1.2, 1.1, 1.5, 0.2, 0.2, 0.5)$ and $K = 50$ for EM algorithm

Simulations



Simulations



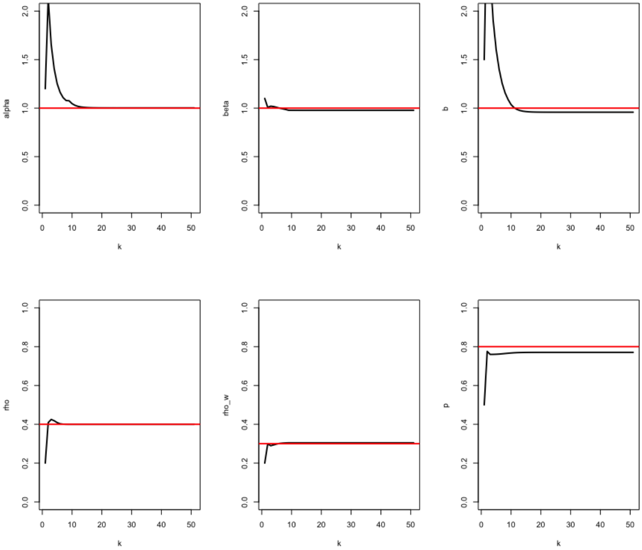
Results

θ	$\theta^\star$	$\hat{\theta}$ $(T = 200)$	$\hat{\theta}$ $(T = 100)$	$\hat{\theta}$ $(T = 50)$	$\hat{\theta}$ $(T = 25)$
α	1	0.689	1.072	1.211	7.085
β	1	1.065	1.046	0.994	0.738
b	1	0.912	1.414	1.220	1.927
ρ	0.2	0.199	0.202	0.156	0.255
ρ_w	0.3	0.292	0.270	0.385	0.037
p	0.7	0.679	0.693	0.682	0.700

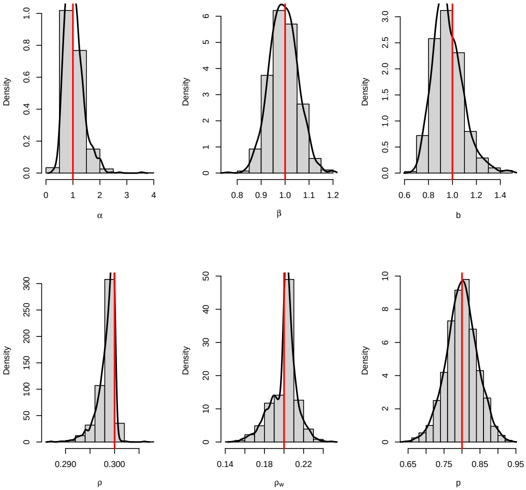
By setting the rule $\tilde{p}_i^K < 0.25$ then ith maintenance is worse,

- for $T = 200$, we find all the worse maintenances and only one false positif.
- for $T = 100$, we find all the worse maintenances and two false positives
- for $T = 50$, we find all the worse maintenances and no false positif
- for $T = 25$, we miss one worse maintenance and one false positif

Convergence of EM algorithm ($T = 200$)



Histogram with 1000 replications ($T = 200$)



To be continued...

- Sensitive analysis of parameters
- Application on a real dataset
- When we observe N systems (with the same parameters)
- Stationary distribution of the Maintained System
- Maintenance optimization with unavailability cost (the failure is only observed at inspection)

Thank you for your attention