

**RELIABILITY DEGRADATION
AND
OPTIMAL MAINTENANCE
FOR INFORMATION EQUIPMENT
INSTALLED ON RAILWAY CARS**

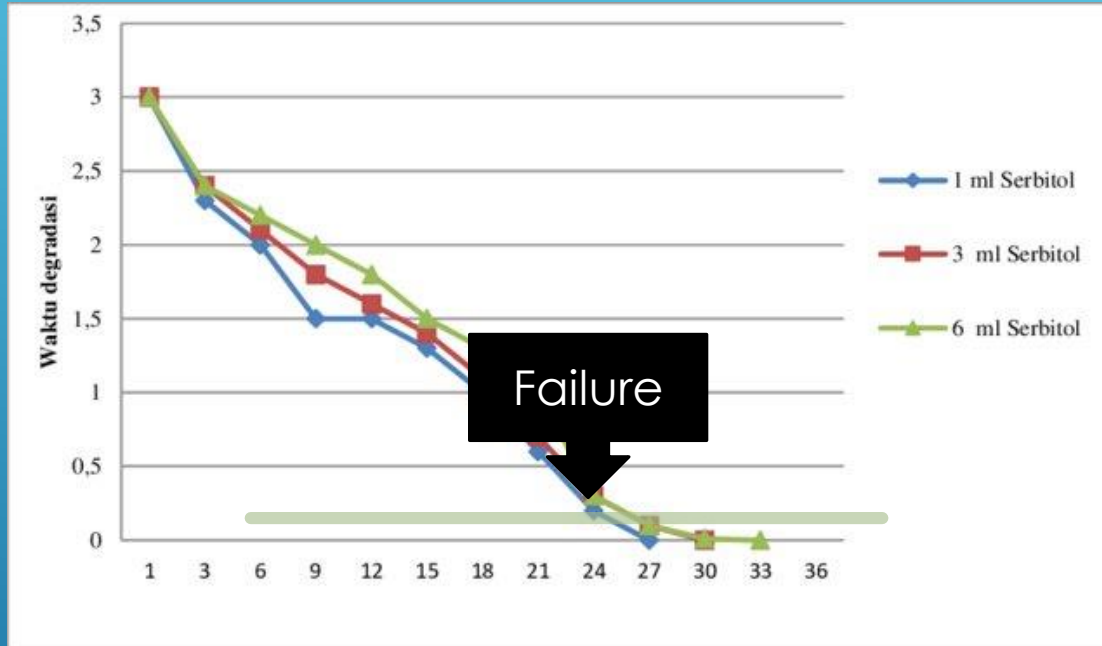


1. **Scope**
2. **Brownian Motion - cause of many degradation processes**
3. **Analyzing Degradation Data**
4. **Resulting Reliability functions**
5. **Maintenance Costs**

Purpose



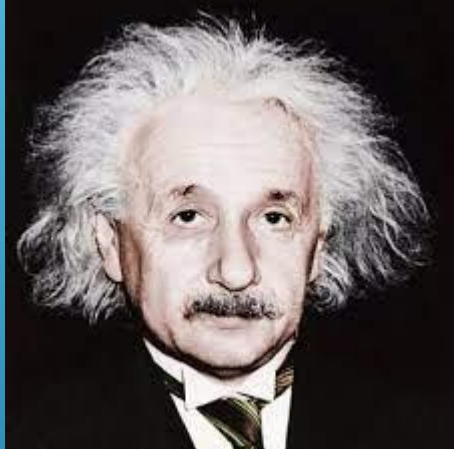
Typical degradation curve



Analyzed : Luminosity of Screen on Railway



Albert Einstein



- ▶ Ph D Thesis : Brownian Motion
- ▶ Noble Prize : photoelectric effect
- ▶ LED's : Reverse Photoelectric Effect

THE RENDEZ-VOUS

- ▶ See Peskir
- ▶ 1905 Einstein : Heat equation (Brownian Motion)
- ▶ 1914 Fokker , 1917 Planck add non constant drift :
- ▶ 1924 Wiener proves the existence of a stochastic process satisfying the Einstein's postulates
- ▶ 1931 – Kolmogorov systematic study of the above forward equation
- ▶

$$p_t = Dp_{xx}$$

$$p_t = -(\mu p)_y + (Dp)_{yy}$$

Ott & Gua

HISTORY OF AN EQUATION

Definition 1. A *standard (one-dimensional) Wiener process* (also called *Brownian motion*) is a stochastic process $\{W_t\}_{t \geq 0+}$ indexed by nonnegative real numbers t with the following properties:

- (1) $W_0 = 0$.
- (2) With probability 1, the function $t \rightarrow W_t$ is continuous in t .
- (3) The process $\{W_t\}_{t \geq 0}$ has *stationary, independent increments*.
- (4) The increment $W_{t+s} - W_s$ has the $\text{NORMAL}(0, t)$ distribution.

A particular case : ▶

Degradation of ▶
normally distributed
parameters

Continuous ▶

Increments :IID, ▶
Normally distributed

WHY BROWNIAN

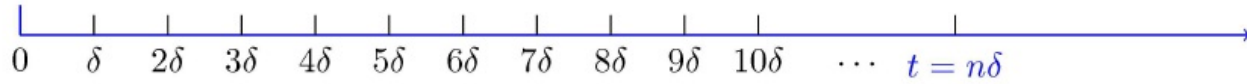


Figure 11.30 - Dividing the half-line $[0, \infty)$ to tiny subintervals of length δ .

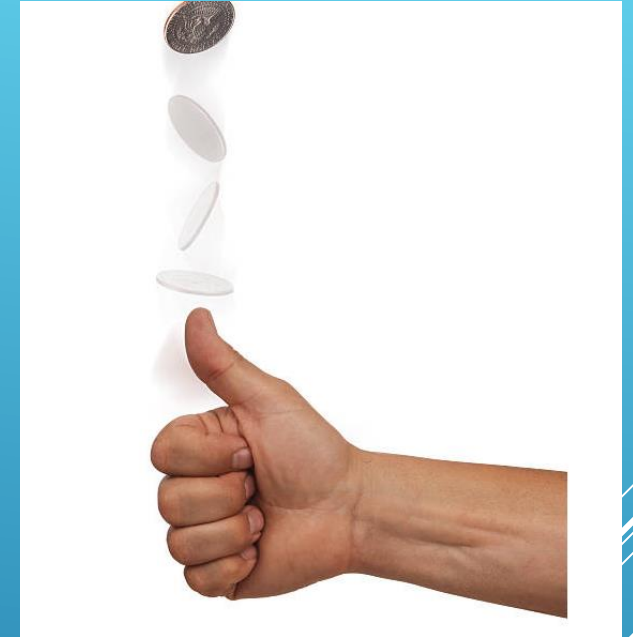
$$X_i = \begin{cases} \sqrt{\delta} & \text{with probability } \frac{1}{2} \\ -\sqrt{\delta} & \text{with probability } \frac{1}{2} \end{cases}$$

$$W(t) = W(n\delta) = \sum_{i=1}^n X_i.$$

$$E[W(t)] = \sum_{i=1}^n E[X_i] = 0,$$

$$\begin{aligned} \text{Var}(W(t)) &= \sum_{i=1}^n \text{Var}(X_i) \\ &= n \text{Var}(X_1) \\ &= n\delta \\ &= t. \end{aligned}$$

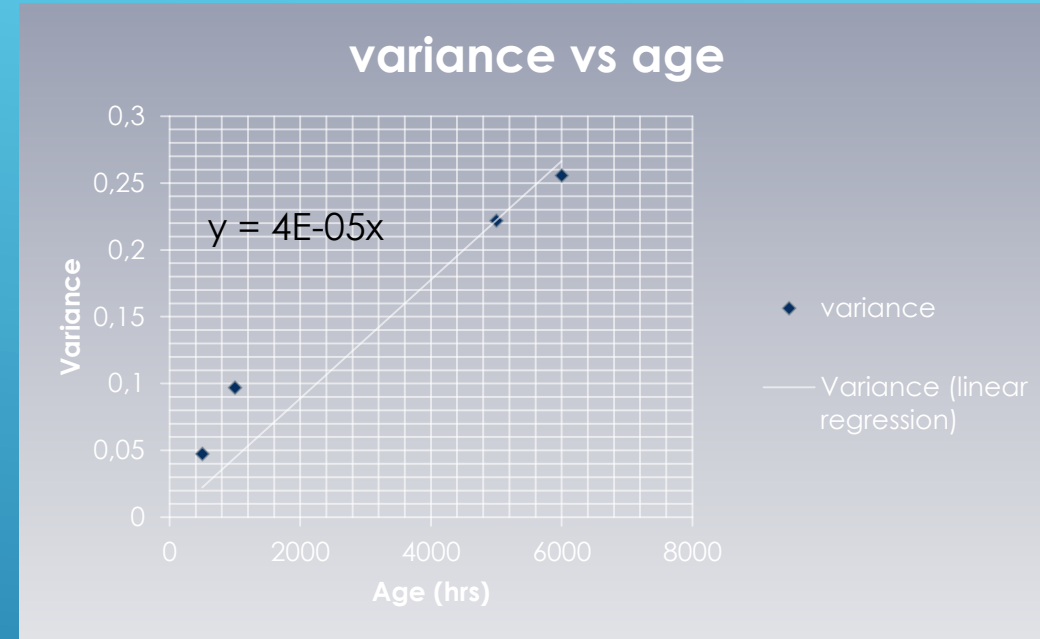
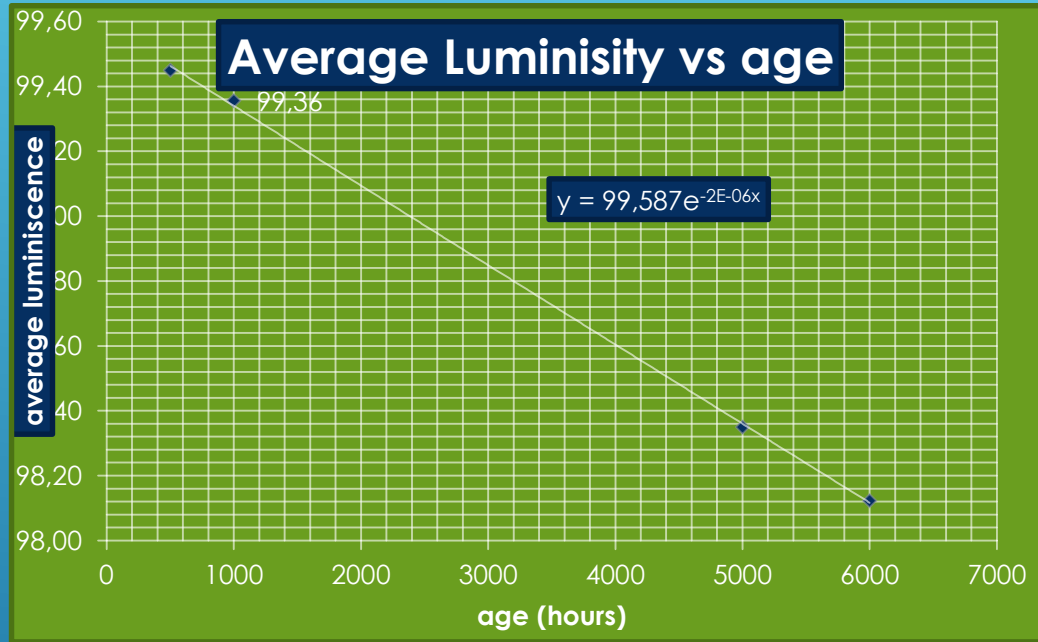
Head / tails



EXPECTATION AND VARIANCE OF A RANDOM WALK

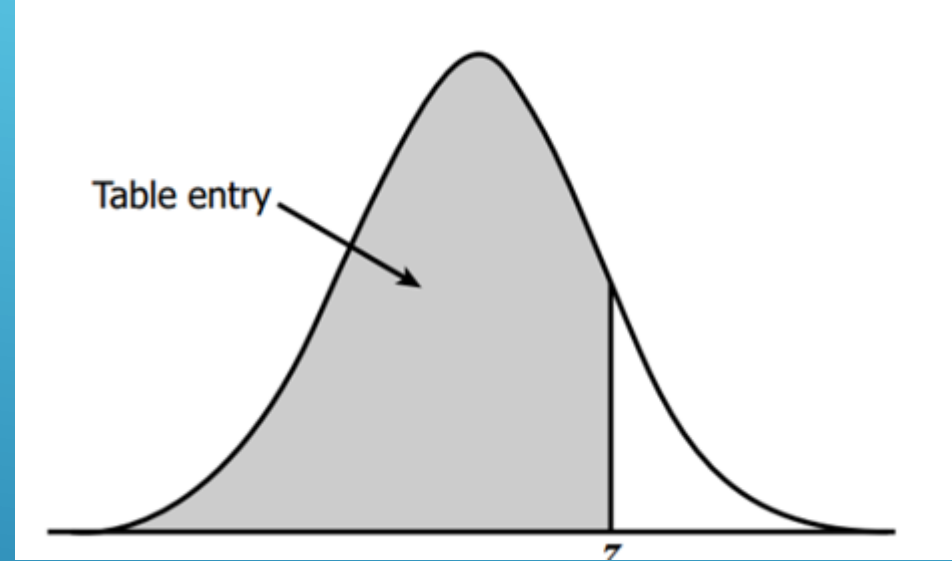
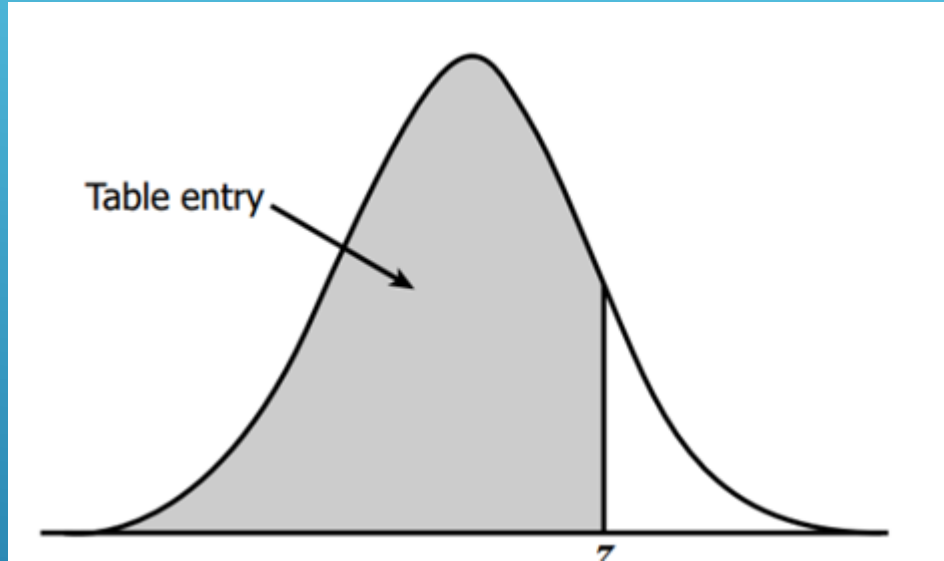
S. E. L. B. quality team, \Report no. : Sled-13-007 Im80 test report," Samsung Electronics

LED Business,Korea, Tech. Rep., 2018.



ANALYSIS OF SAMSUNG'S REPORT

Approaches by average Ott,Guo

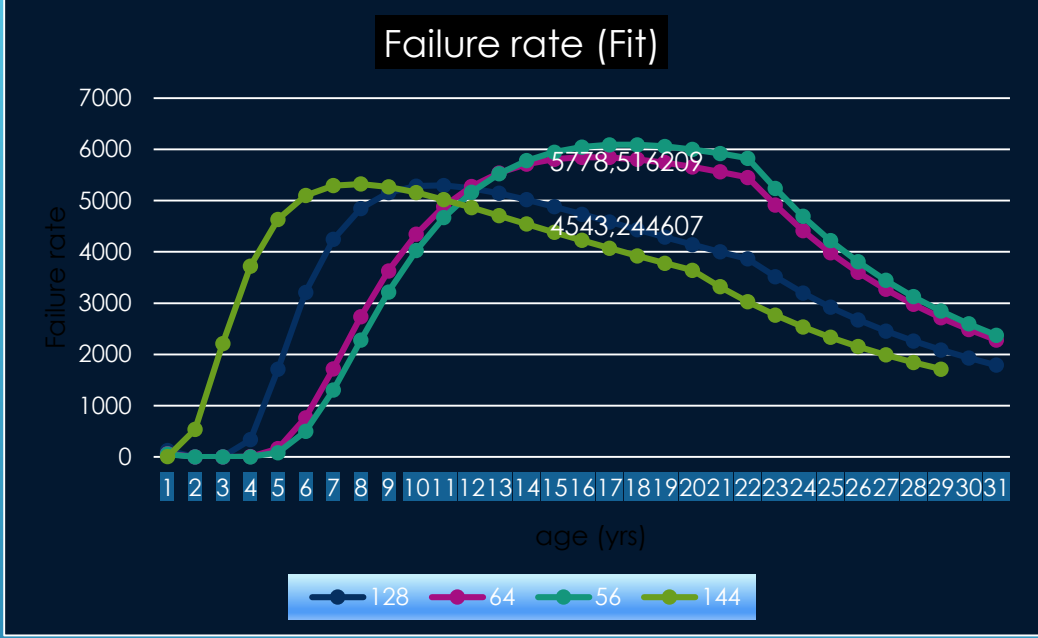
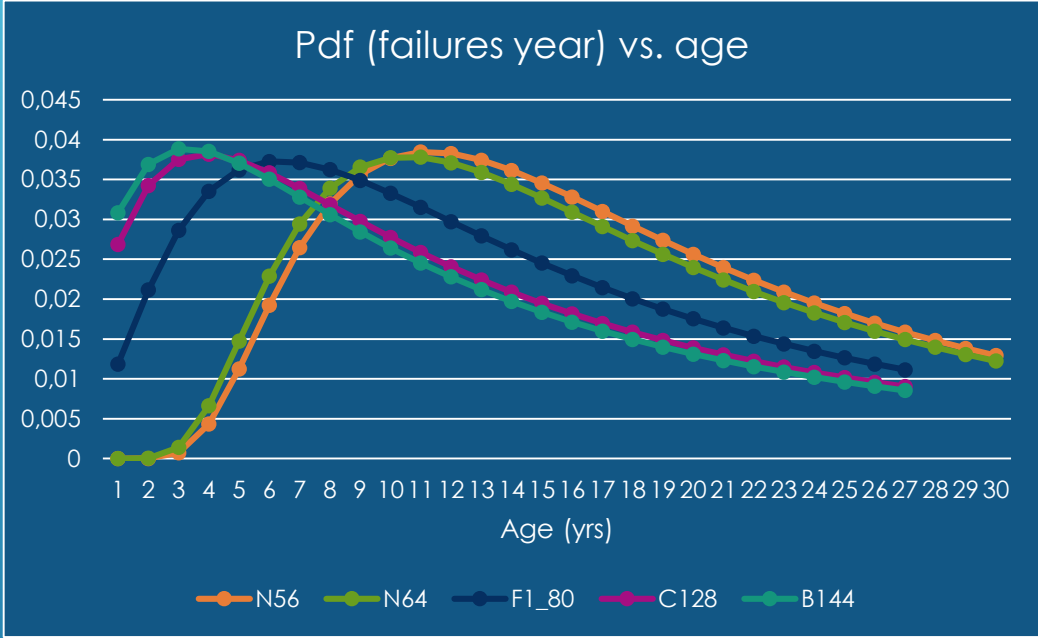


Z - SCORE

RELIABILITY FUNCTIONS

yrs	zN	R(t)	cdf	pdf	FR
1	6.272566324	1	1.77573E-10	1.77573E-10	1.77573E-10
2	4.164280423	0.999984383	1.56168E-05	1.56166E-05	1.56169E-05
3	3.182617828	0.999270249	0.000729751	0.000714134	0.000714655
4	2.571136232	0.994931728	0.005068272	0.004338522	0.004360622
5	2.13701834	0.98370175	0.01629825	0.011229978	0.011416039
6	1.80489899	0.964454755	0.035545245	0.019246995	0.019956348
7	1.538259693	0.938007435	0.061992565	0.02644732	0.028195214
8	1.316888649	0.906062024	0.093937976	0.03194541	0.035257421
9	1.128529518	0.870451821	0.129548179	0.035610203	0.040910022
10	0.965235744	0.832786626	0.167213374	0.037665195	0.045227905
11	0.821584015	0.794343155	0.205656845	0.038443471	0.048396554
12	0.693719686	0.756070991	0.243929009	0.038272164	0.050619802
13	0.578811178	0.718641706	0.281358294	0.037429285	0.052083374
14	0.474720788	0.682507001	0.317492999	0.036134705	0.05294408
15	0.379797038	0.647951959	0.352048041	0.034555041	0.053329635
16	0.292738712	0.615139061	0.384860939	0.032812898	0.053342245
17	0.21250297	0.584142669	0.415857331	0.030996392	0.05306305
18	0.138241532	0.554975234	0.445024766	0.029167436	0.052556284
19	0.069255288	0.527606792	0.472393208	0.027368441	0.051872799

Xiao-sheng 's approach



MAINTENANCE & SPARE STORE



Definition: $U_j(t)$

- ▶ $U_j(t)$ = Prob of j or more failures until t
- ▶ $U_1(t) = 1 - R(t)$

$$r(t) = \sum_{i=1}^n U_i(t)$$

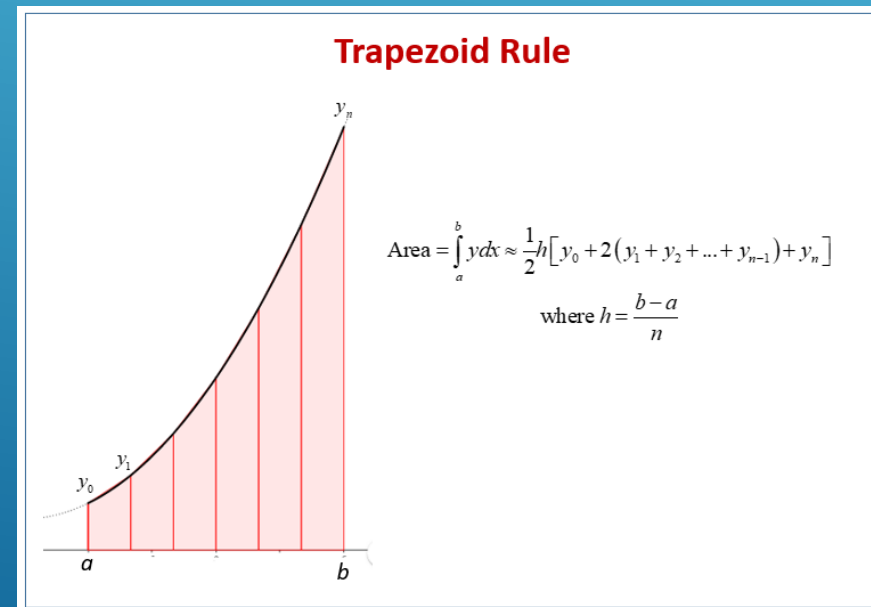
Iterative calculation of $U_j(t)$ Feller

$$U_j(t) = \int_{\tau=0}^t U_{j-1}(t-\tau) f(\tau) d\tau$$

Convolution

Integral evaluated by trapezoidal rule

n must be large enough to ensure quasi linearity of the integrand



AVERAGE NO OF FAILURES IN AN INTERVAL

Correction for Non Linearity

For small fleets a correction is needed because λ is not constant and not linear

$$\gamma = TAT/\Delta t$$

$$\text{Use } D = Nr(t)\left(\gamma + \frac{\sqrt{\gamma(1/4 + \epsilon\%)}}{\sqrt{N}}\right)$$

For large fleets ($N \rightarrow Q_p$) use :

$$D = Nr(t) \gamma$$

Poisson For each Δt

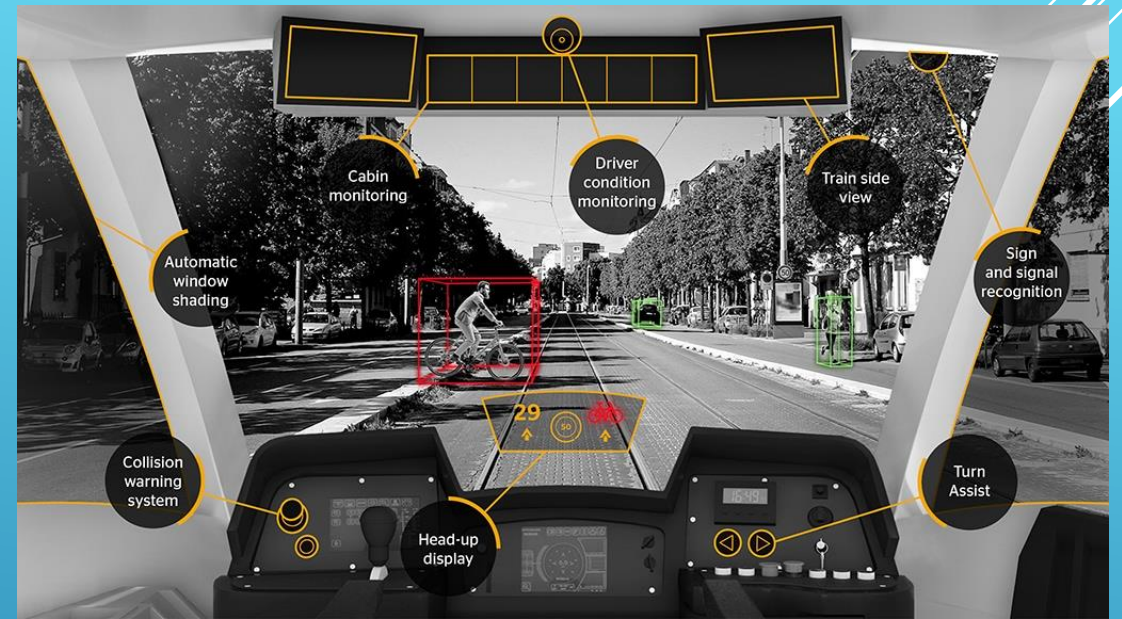
Find k , so that :

$$P(D) = \sum_{j=1}^k \frac{D^j}{j!} e^{-D} < 1 - \epsilon$$

RECOMMENDED SPARE PARTS



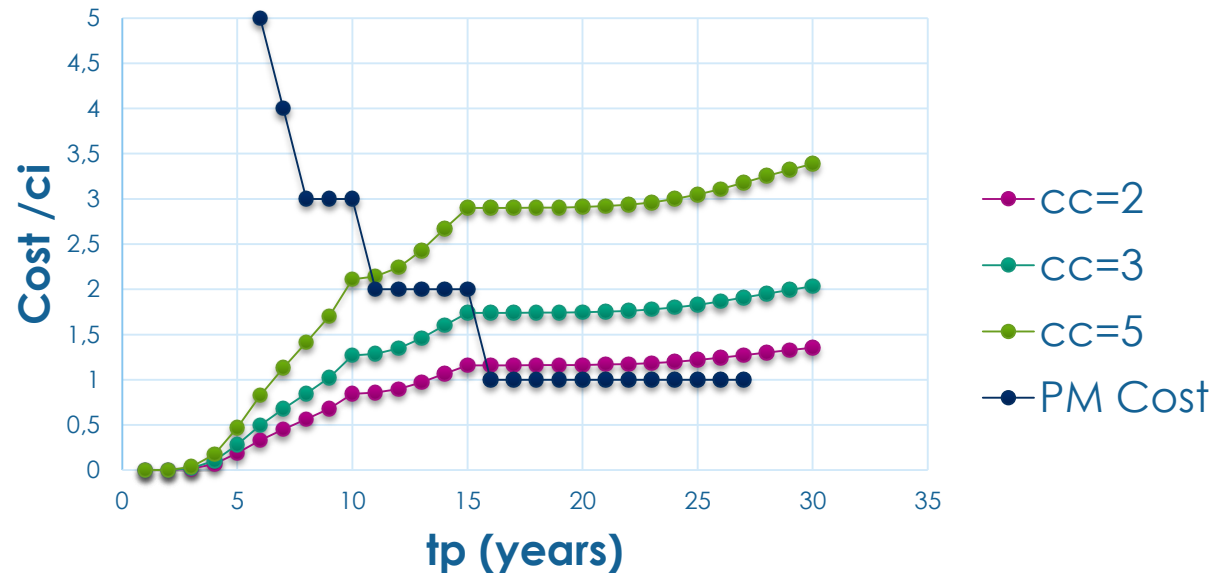
Info system



Head up display

Optimal Maintenance

Cost of unav vs tp



Optimal Maintenance

DEFINITIONS

$t_p \equiv$ time for initiated replacement

$$n \equiv \text{Int}\left(\frac{t_L}{t_p}\right)$$

$$t_r \equiv \text{mod}(t_L, t_p)$$

$F(t) \equiv 1 - R(t)$, prob of failure, with no initiated replacement

$F_p(t) \equiv 1 - R_p(t)$, prob of failure, with initiated replacement

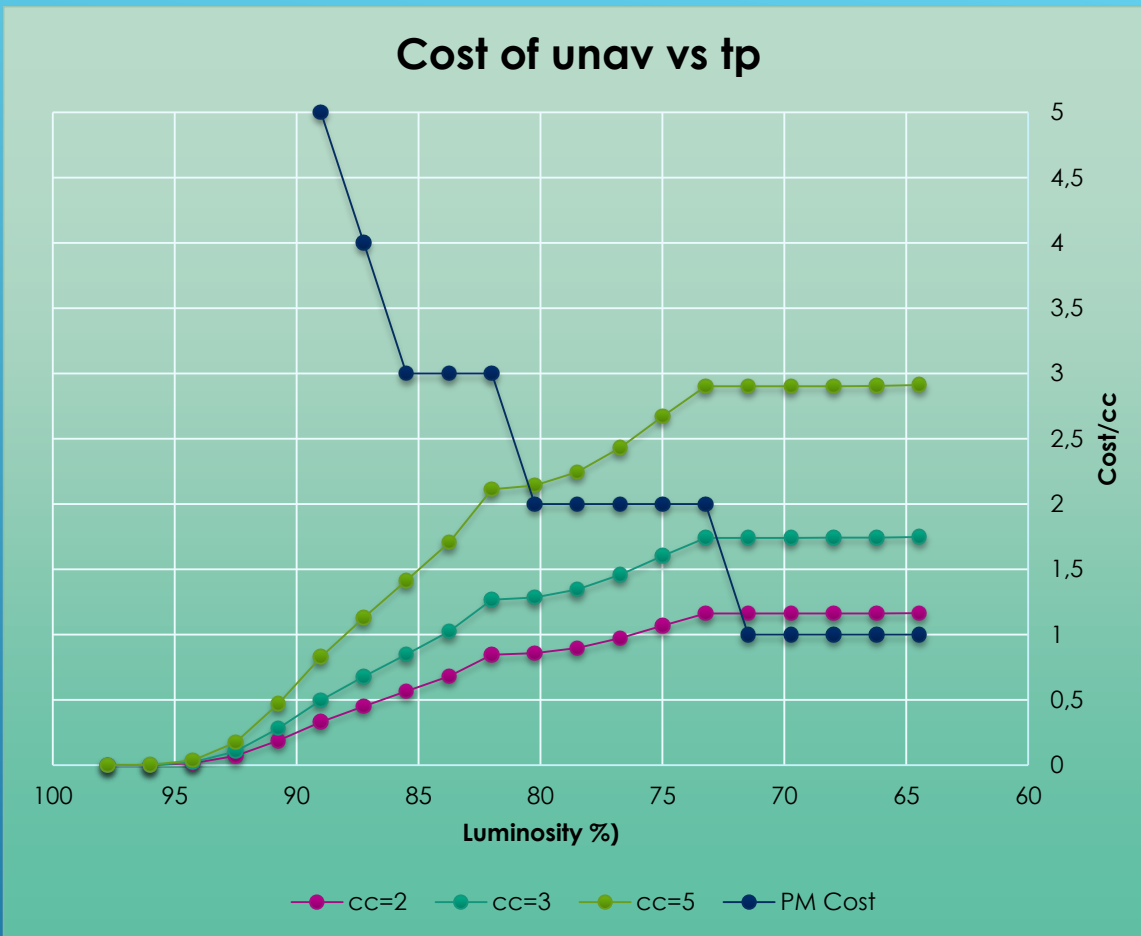
$ci \equiv$ cost of an initiated replacement

$cu \equiv$ cost unavailability, given it happened

$CU \equiv$ expected unavailability cost

$CI \equiv$ total cost of initiated replacement during t_L

$$cc \equiv \frac{cu}{ci}$$



If on the horizontal line we replace time, by the relative loss in luminosity, we get a similar graph, but the decision point on optimal maintenance is by luminosity instead of time

The advantage: The decision on replacement of the component takes in account the individual history of the component (production lot, environment, actual use etc.)

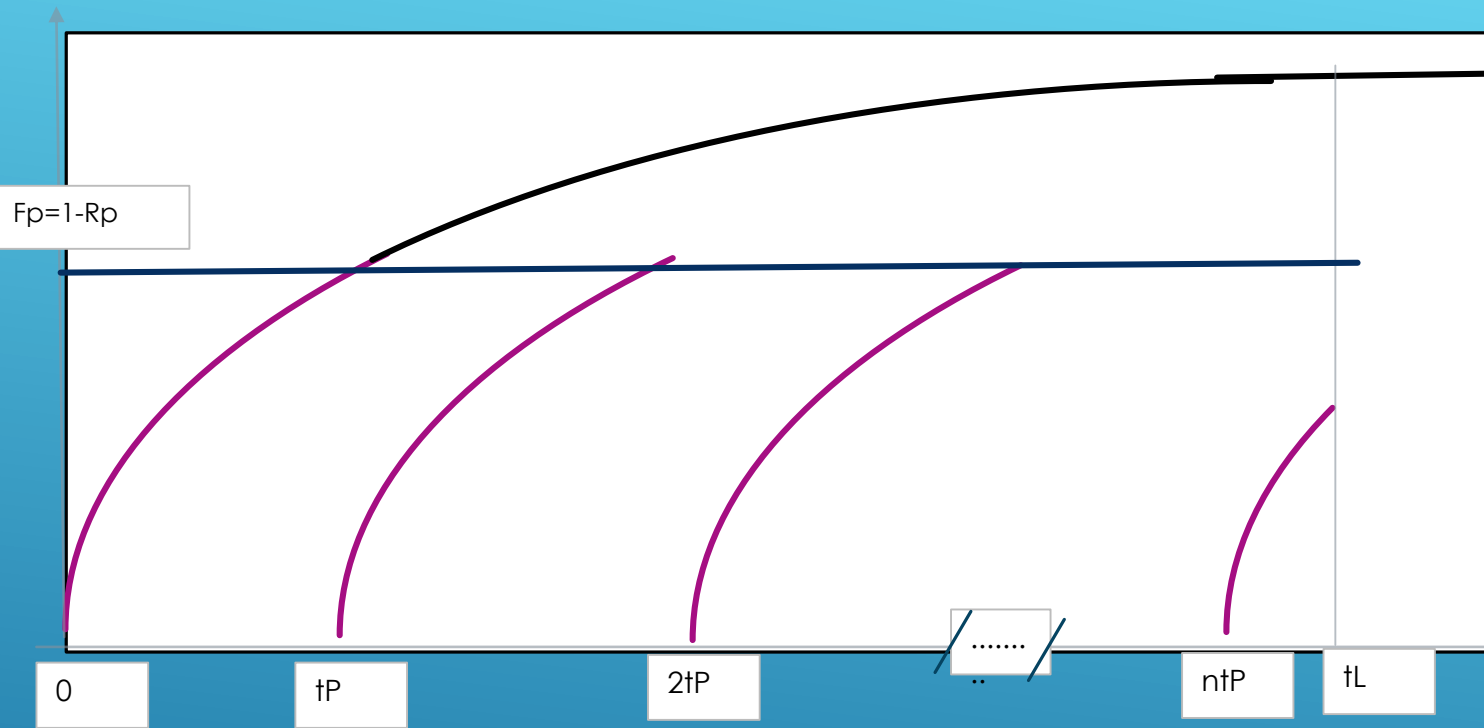
The cost : Need for periodic luminosity measurement

If the gain from the advantage is higher than the cost – the individual monitoring is beneficial.

Optimal Maintenance individual monitoring

1. Xiao-sheng Si, Changhua hu, . Wenbin Wang, M. Pecht *Remaining Useful Life Estimation Based on a Nonlinear Diffusion Degradation Process*, IEEE Transactions on Reliability - March 2012
2. Schneider, Eric D., and James J. Kay "Life as a manifestation of the second law of thermodynamics." Mathematical and computer modelling 19.6-8 (1994): 25-48.
3. Peskir, Goran. "On boundary behaviour of one-dimensional diffusions: from Brown to Feller and beyond."
4. William Feller, Select
5. H. Pishro-Nik, "Introduction to probability, statistics, and random processes", available at <https://www.probabilitycourse.com>, Kappa Research LLC, 2014.
6. LIVNI, Haim. Life cycle maintenance costs for a non-exponential component. *Applied Mathematical Modelling*, 2022, 103: 261-286.
7. M. Ott, "Capabilites and Reliability of LEDs and Laser Diodes," NASA Goddard Space Flight Center, 9 March 97 { Online}. Available https://www.researchgate.net/publication/2915483_Capabilites_and_Reliability_of_LEDs_and_Laser_Diod
8. Guo. Liao, " Practical Approaches for Reliability Evaluation Using Degradation Data," in *Reliability and Maintainability Symposium*, Tucson, Arizona, January,

REFERENCES



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CALCULATIONS

$$F_p(t_L) = F(t_p) + R(t_p)F(t_p) + \dots + \dots R(t_p)^n F(t_p) + R(t_p)^n F(t_r)$$

$$F_p(t_L) = \cancel{F(t_p)} \frac{1 - R(t_p)^n}{\cancel{F(t_p)}} + R(t_p)^n F(t_r)$$

$$CU = F_p(t_L) [(1 - R(t_p)^n) + R(t_p)^n * F(t_r)] * cc * ci$$

$$CI \equiv n * cu$$