Contribution ID: **16** Type: **not specified**

Optimal design to test for heteroschedasticity in a regression model

Monday, June 27, 2022 1:50 PM (20 minutes)

The goal of this study is to design an experiment to detect a specific kind of heteroscedasticity in a non-linear regression model, i.e.
$y_i = \eta(x_i;\beta) + \varepsilon_i,\ \varepsilon_i \sim N(0;\sigma^2 h(x_i;\gamma)), \quad i = 1,\dots,n, where \eta(x_i;\beta)$ is a non-linear mean function, depending on a vector of regression coefficients $\beta \in \mathbb{R}^m$, and $\sigma^2 h(x_i;\gamma)$ is the error variance depending on an unknown constant σ^2 and on a continuous positive function $h(\cdot;\cdot)$, completely known except for a parameter vector $\gamma \in \mathbb{R}^s$.

In many practical problems, it may be meaningful to test for the heteroscedasticity, that is to consider the null hypothesis $H_0:\ \gamma = \gamma_0$, where γ_0 is a specific value leading to the homoscedastic model, i.e. $h(x_i;\gamma_0) = 1$, and a local alternative $H_1:\ \gamma = \gamma_0 + \lambda/\sqrt{n}$, with $\lambda \neq 0$. The application of a likelihood-based test (such as log-likelihood ratio, score or Wald statistics) is a common approach to tackle this problem, since its asymptotic distribution is known.

The aim of this study consists in designing an experiment with the goal of maximizing (in some sense) the asymptotic power of a likelihood-based test. The majority of the literature in optimal design of experiments concerns the inferential issue of precise parameter estimation. Few papers are related to hypothesis testing. See for instance, Stigler (1971), Spruill (1990), Dette and Titoff (2009) and the references therein, which essentially concern designing to check an adequate fit to the true mean function. In this study, instead, we justify the use of the D_s-criterion and the KL-optimality (Lopez-Fidalgo, Tommasi, Trandafir, 2007) to design an experiment with the inferential goal of checking for heteroscedasticity. Both D_s- and KL-criteria are proved to be related to the noncentrality parameter of the asymptotic chi-squared distribution of a likelihood test.

Keywords

Optimal discrimination designs; asymptotic power; likelihood test

Primary author: TOMMASI, chiara (University of Milan)

Co-authors: Dr LANTERI, Alessandro (University of Milan); Prof. LEORATO, Samantha (University of Milan)

Presenter: TOMMASI, chiara (University of Milan)

Session Classification: CONTRIBUTED Design of Experiment 1

Track Classification: Design and analysis of experiments