Statistical Modeling and Monitoring of Geometrical Deviations in Complex Shapes

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Motivating Example: Control of Complex Parts

Scimone et al. (2021)

Metal Additive Manufacturing Productions

- a) lightweight bracket for space applications
- b) topologically optimized space antenna support
- c) rocket engine demonstrator

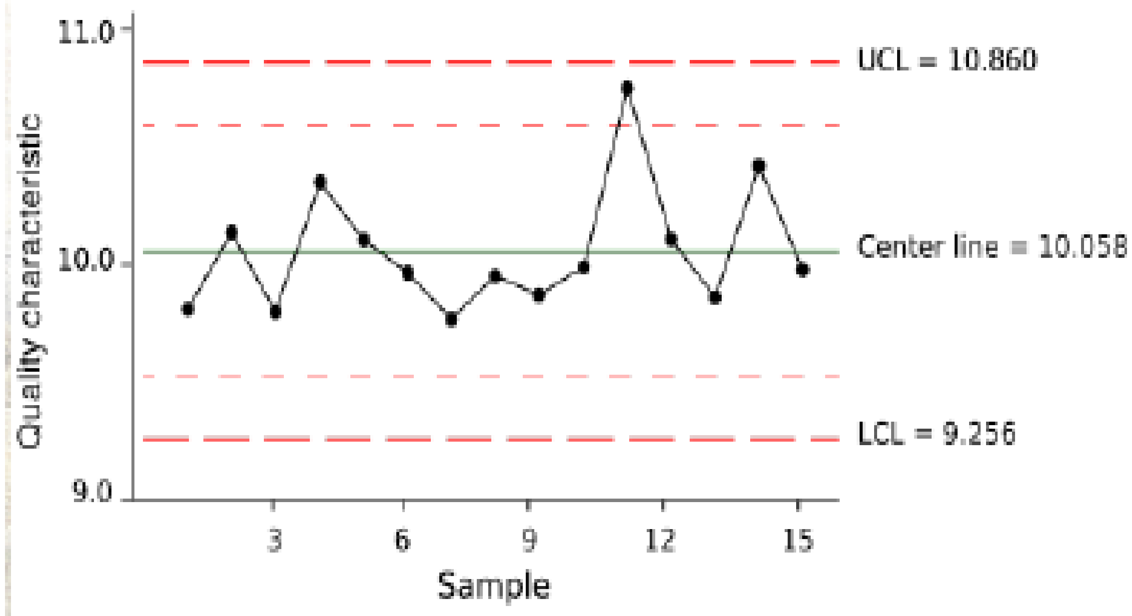
- Geometries cannot be easy parametrized
- Statistical Process Control is not straightforward

From Simple to Complex Shapes

Scimone et al. (2021)

Clear Quality Features

Simple Control Charts

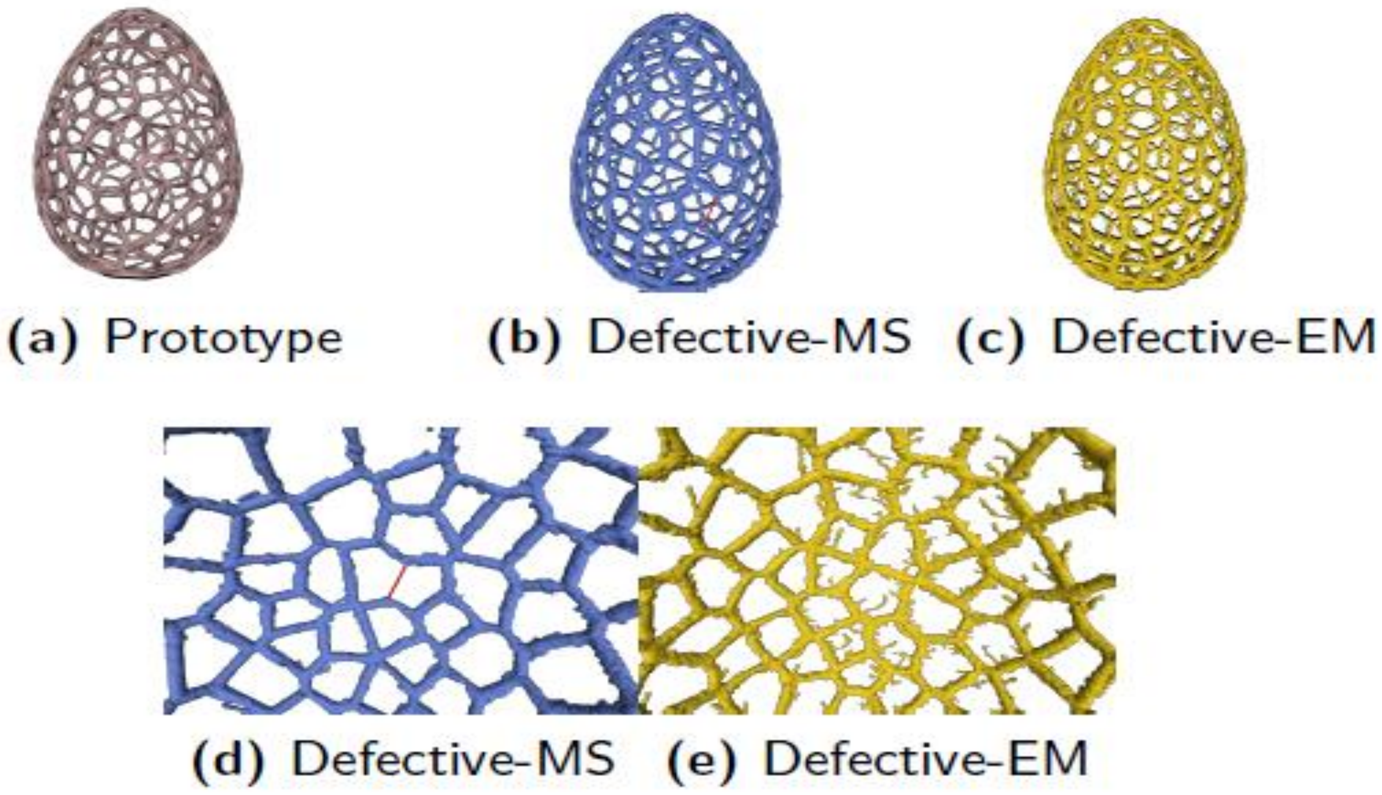


- For simple parts as screw and bolts, quality features are straightforward to identify, and uni- or multi-variate control charts can be built!

From Simple to Complex Shapes

Scimone et al. (2021)

What Quality Features?

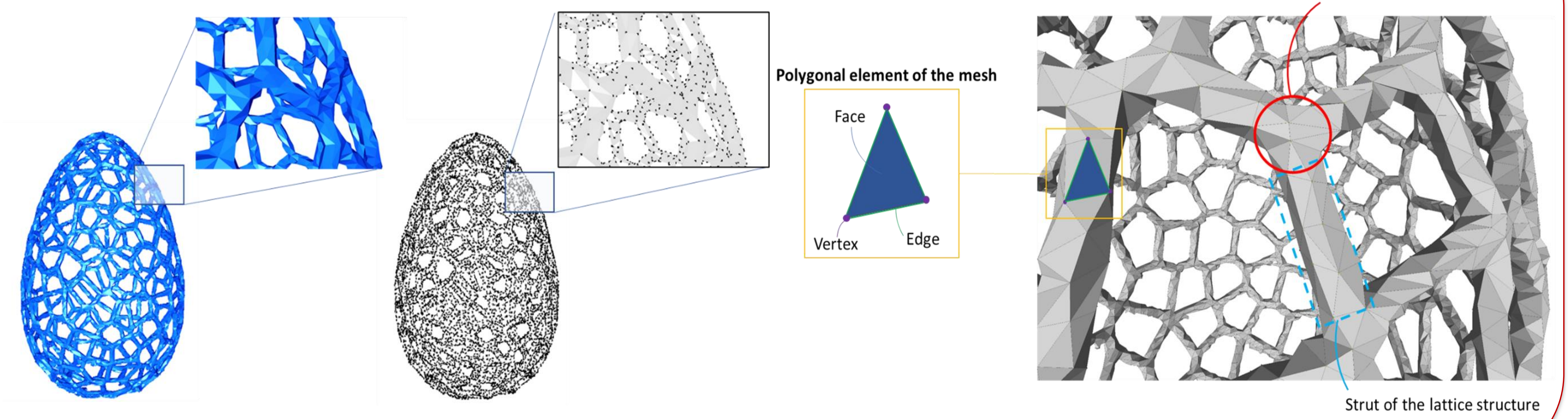


- **The egg shells were manufactured via AM at Dipartimento di Meccanica, Politecnico di Milano**

From Complex Objects to Complex Data

Scimone et al. (2021)

Mesh and Point Cloud Data



- Mesh and point cloud data are obtained by X-ray Computed Tomography on the manufactured shapes
- N = 16 objects were manufactured
- Our data consists of a prototype mesh P and real objects meshes $S_j, \quad j = 1, \dots N$

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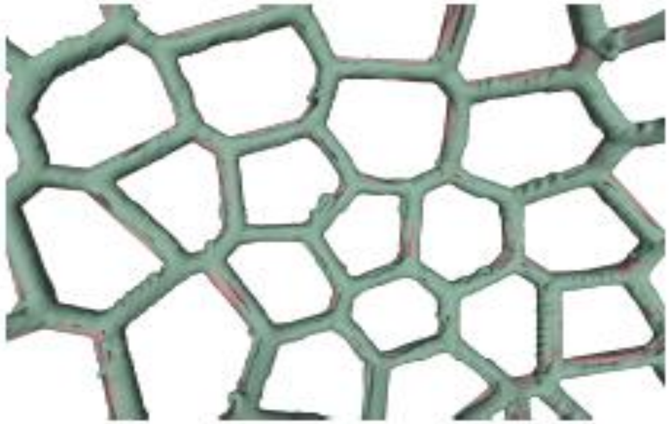
Modeling Geometric Deviations: Prototype vs Real Objects

Scimone et al. (2021)

Mesh and Point Cloud Data

(a) Nominal Model mesh



(b) Geometrical mismatch after ICP algorithm

(c) Printed mesh

- **How should we capture all information about geometric deviations?**

Modeling Geometric Deviations: Hausdorff Distance

Scimone et al. (2021)

Natural metric between sets

With P we denote the prototype, with S_j we denote the j-th mesh. $P, S_j \subset \mathbb{R}^3$

- $d_{S_j}(p) \coloneqq \min_{s \in S_j} d\,(p, s)\ \forall\ p \in P$ Deviation map between P and S_j
- $d_P^j(s) \coloneqq \min_{p \in P} d\,(s, p)\ \forall\ s \in S$ Deviation map between S_j and P
- $d_H\left(P, S_j\right) := \max\left\{max_{p \in P}\ d_{S_j}(p), max_{s \in S}\ d_P^j(s)\right\}$ Hausdorff Distance

Metric between subsets of a metric space, naturally induced by the metric space itself

Hausdorff Distance and defect characterization

Scimone et al. (2021)

Defect characterization

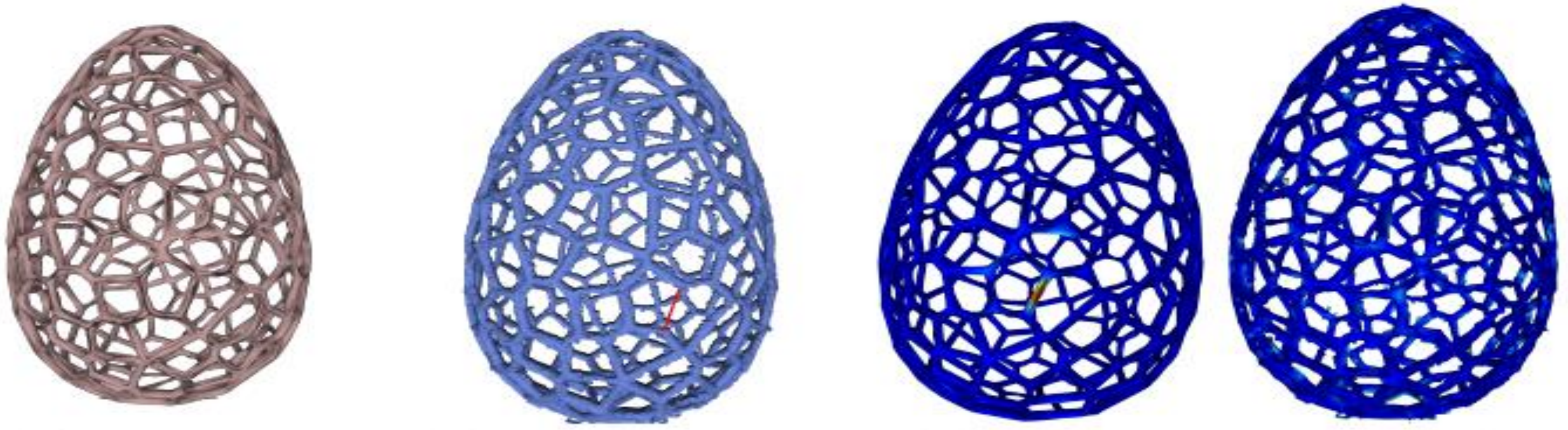
Since $d_H(P, S_j) = 0 \Leftrightarrow P = S_j$, the couple of maps (d_P^j, d_{S_j}) fully characterizes the geometrical differences between the object and the prototype.

- The two maps generally carry different and complementary information
- In previous works, where simple objects and defects were considered, only one deviation map is analysed (Wells et al., 2013, and reference therein), **but we need both for a complete characterization!**
- The deviation maps are spatial functions with a different 3D domain. Moreover, the d_P^j have different domains and cannot be directly compared

Example: Complementariety of the Maps

Scimone et al. (2021)

Deviation Maps, missing srut



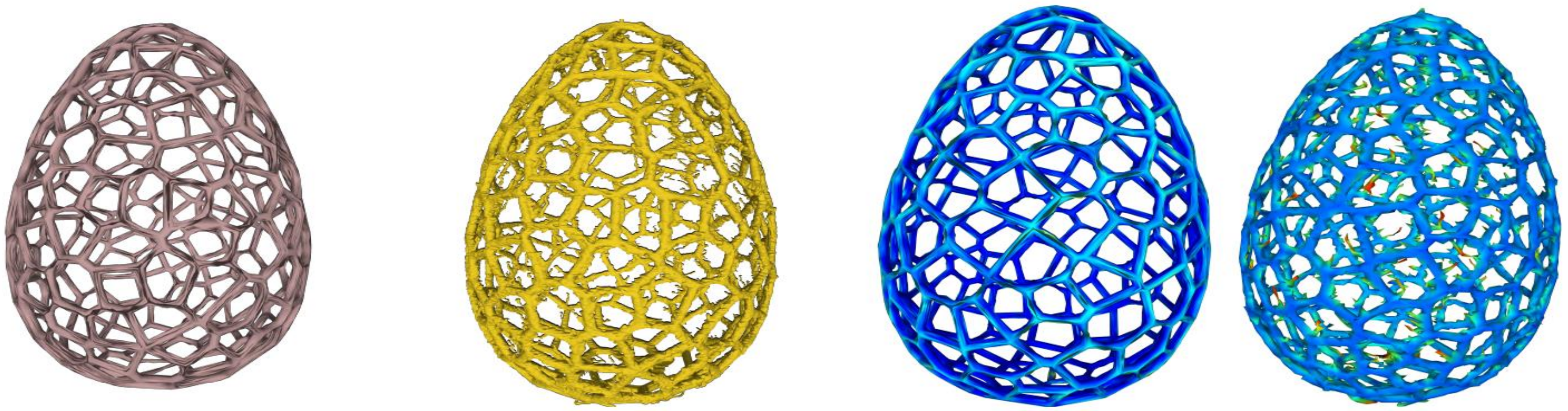
(a) Prototype (b) Defective-MS (c) map d_{S_j} (d) map d_P^j

- d_P^j: $S_j \to \mathbb{R}^3$, $d_P^j(s) := min_{p \in P}\, d(s,p)$ **cannot see the defect in this case (no points associated to high values of distance)**
- d_{S_j}: $S_j \to \mathbb{R}^3$, $d_{S_j}(p) := min_{s \in S_j}\, d(p,s)$ **can!**

Example: Complementariety of the Maps

Scimone et al. (2021)

Deviation Maps, missing srut



(a) Prototype (b) Defective-EM (c) map d_{S_j} (d) map d_P^j

- **Here the situation is reverted!**

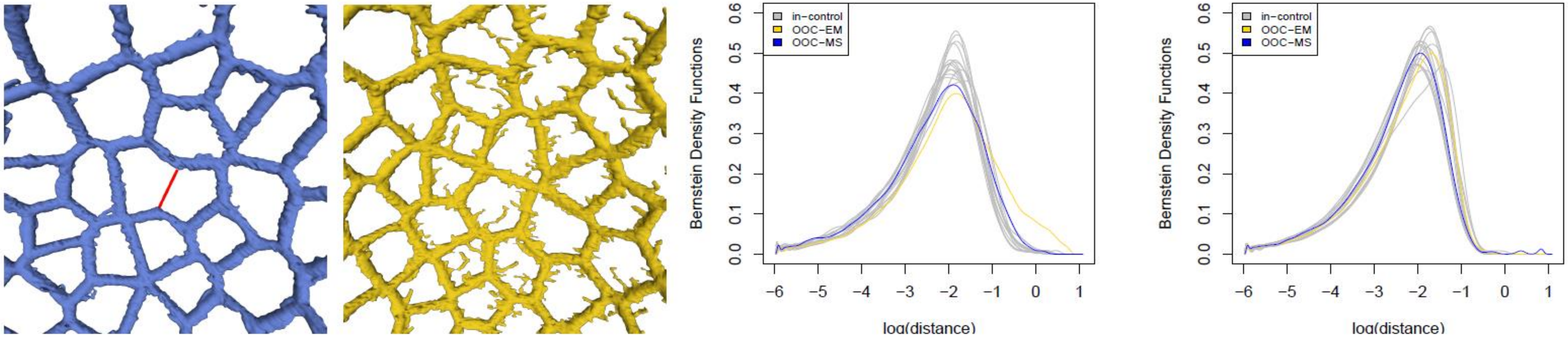
From Distance Maps to Densities

Scimone et al. (2021)

Summarizing maps for proper comparison

- $d_{S_j} \rightarrow f_{S_j}$, density of distances of points of P from S_j
- $d_P^j \rightarrow f_P^j$, density of distances of points of S_j from P
- Two N-dimensional datasets, $f_S := \{f_{S_j}\}_{j=1,\dots,N}$ and $f_P := \{f_P^j\}_{j=1,\dots,N}$ with a precise geometric interpretation

From N objects to 2N densities



(a) OOC-MS (b) OOC-EM (c) f_P (d) f_S

- **The initial dataset of 16 trabecular egg shells has been represented by two datasets of probability densities. Densities are estimated via Bernstein polynomials**

Why Densities?

Scimone et al. (2021)

Natural Choice

- Natural extension of what is done in previous works (study of moments, QQ-plots)
- The mathematical theory is solid enough to extend SPC tools as control charts

Quick recap on B^2 geometry

$$B^2(\Omega) := \{f > 0 \; on \; \Omega, \; log(f) \in L^2(\Omega), f_1 := f_2 \; iff \; f_1 = \alpha f_2 \}$$

It's an Hilbert space with appropriate operations between densities. Only ratios between parts matters, in a compositional fashion

- $f_1 + f_2 := \frac{f_1 f_2}{\int_\Omega f_1 f_2}$
- $\beta \cdot f_1 := \frac{f_1^\beta}{\int_\Omega f_1^\beta}$
- $\langle f_1, f_2 \rangle = \frac{1}{2|\Omega|} \int_\Omega \int_\Omega log \frac{f(t)}{f(s)} log \frac{g(t)}{g(s)} \, dtds$

Control framework: Profile Monitoring of Density Functions

Scimone et al. (2021)

Summarizing a daset of densities

For control, we need to build appropriate statistics:

- PCA is consistently extended to Hilbert spaces and thus to B^2 (SFPCA, Hron et al., 2016)
- Standard PCA-based control can then be applied

Computing Scores

Let $(H, +, \langle\cdot,\cdot\rangle)$ be Hilbert, $\{X_i\}_{i=1,\dots,N}$ a dataset with zero mean and sample covariance Σ, that is $\Sigma u = 1/N \sum_i \langle X_i, u\rangle X_i \forall u$. Let $\left(\lambda_j, \zeta_j\right)_{j=1,\dots N}$ be the spectral decomposition of Σ, and $z_{ij} = \langle X_i, \zeta_j\rangle$ the scores. Fix $K \in \{1, \dots, N-1\}$ suitably.

Control framework: Profile Monitoring of Density Functions

Scimone et al. (2021)

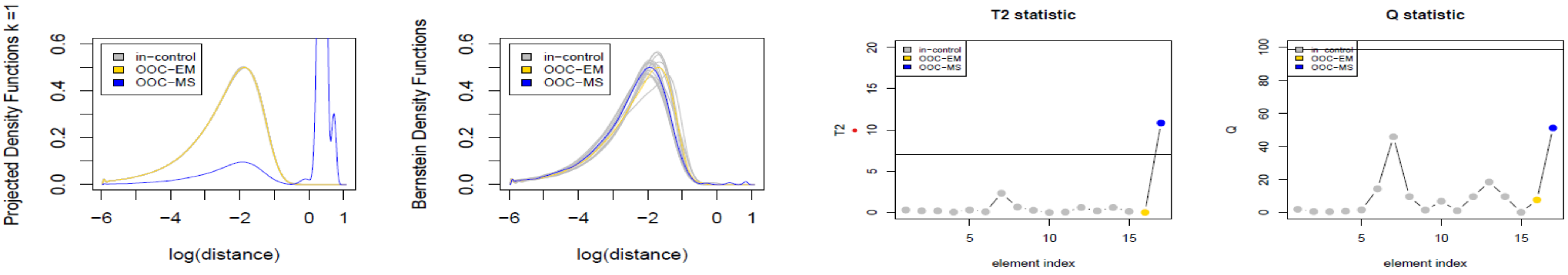
PCA-based statistics for control

- $T_i^2 = \sum_{k=1}^{K} z_{ij}^2/\lambda_j$ measures the distance between the mean and the reconstruction of X_i on the K-th principal subspace $span(\zeta_1, \dots \zeta_K)$, taking into account the data variability
- $Q = \sum_{j=K+1}^{N} z_{ij}^2$ measures the Euclidean distance between the mean and the part of X_i outside the first K-th principal subdpace
- T^2 and Q are uncorrelated and can be used for control, as classical PCA-based control charts

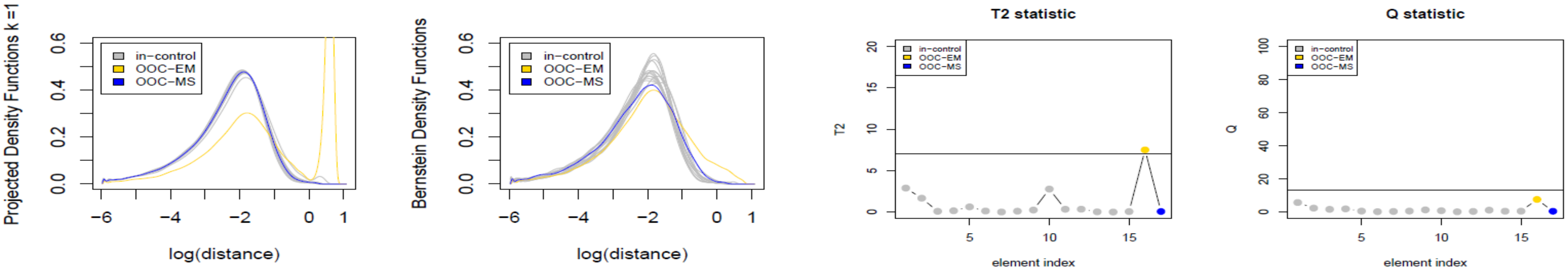
Control charts construction on real data

Scimone et al. (2021)

Control charts on f_{S_j} densities



Control charts on f_P^j densities



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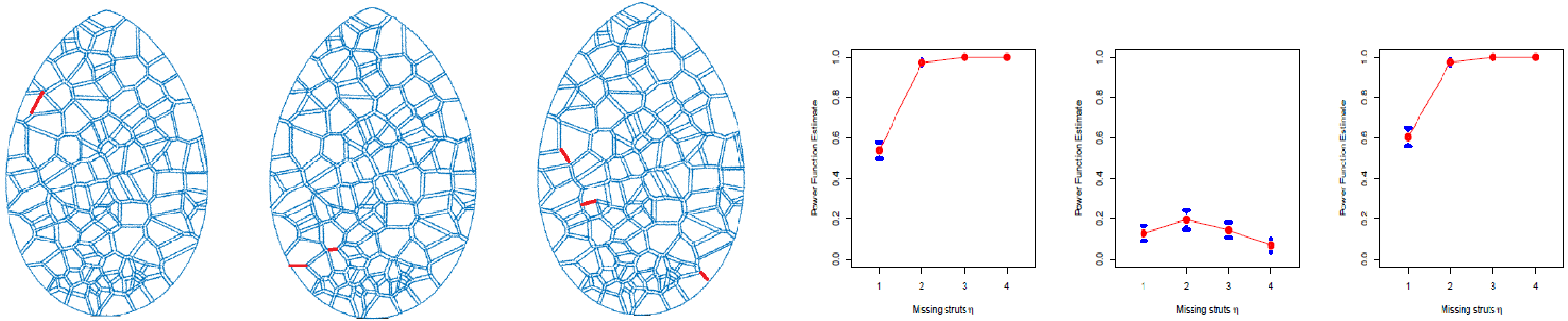
Simulations for Power Estimation

Scimone et al. (2021)

Simulated defectiveness scenarios



(a) 1 strut missing (b) 2 strut missing (c) 3 strut missing (d) f_{S_j} analysis (e) f_P^j analysis (f) Overall power

➢ **Other scenarios were explored, with very satisfactory results**

Conclusion: Final Remarks

Scimone et al. (2021)

A general strategy

The choice of shape descriptors, based on Hausdorff distance, in conjunction with the theory of Hilbert spaces, allow us to:

- Build a general framework for SPC on dataset of scanned objects, regardless of their complexity or topological richness
- Summarize the ``defective'' or ``conformal'' status of an object on the basis of simple statistics
- Design extensive simulation studies
- Detect both widespread and very local defectiveness sources

Some references

- **Scimone, R., Taormina, T., Colosimo, B.M., Grasso, M.L., Menafoglio, A., Secchi, P. (2021): Statistical modeling and monitoring of geometrical deviations in complex shapes with application to Additive Manufacturing, *Technometrics, DOI: 10.1080/00401706.2021.1961870***
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