A PREDICTIVE MAINTENANCE COST-MODEL

FLORIAN SOBIECZKY, IVO BUKOVSKY, ONDŘEJ BUDÍK, MAQBOOL KHAN

Abstract. The benefit of predictive maintenance (PdM) as an enterprise strategy for scheduling repairs compared to other maintenance strategies relies heavily on the optimal use of resources, especially for SMEs: Expertise in the production process, Machine Learning Know-How, Data Quality and Sufficiency, and User Acceptance of the AI-Models have shown to be significant factors in the profit calculation. Using a stochastic model for the production cycle, we show how all these factors determine reduction of revenue and increase in maintenance cost, essentially leading to a quantitative condition for the beneficial use of PdM.

1. Problem

1.1. **Predictive Maintenance.** Predictive maintenance is the strategy of scheduling maintenance times in accordance with the likely remaining useful lifetime of machine parts. Other, more conventional maintenance strategies include preventive maintenance, in which a fixed schedule of maintenance times are set to prevent premature breakdown [5].

1.2. **Related Work.** General approaches to finding the most cost-effective use of predictive maintenance have been reviewed by [2]. More specifically, production environments to which renewal theoretic concepts can be applied have been considered [1]. Specific applications of a cost-model driven use of maintenance is given in energy management [3]. A versatile stochastic model has been developed for preventive maintenance in [4], to which this approach can be seen as an extension to include predictive maintenance and the duration of each complete production cycle as a specific random variable.

1.3. **Organization of this paper.** In section 2 we present the definition of the cost-model and its general implications for the choice of the maintenance strategy. In section 3 several experimental scenarios are discussed, and Section 4 presents the conclusion. The notation used in this article includes $\mathbb{R}_+$ denoting the non-negative numbers,

2. Solution

2.1. **A Maintenance Strategy Cost-Model.** We first provide the necessary stochastic model. Consider a probability space $(\Omega, \mathcal{F}, \mu)$ sufficiently large for the random variables $\tau_i, \rho_i : \Omega \to \mathbb{R}_+, i \in \mathbb{N}$, to represent proper real valued random variables for the *scheduled maintenance time*, and *repair time* for the i'th maintenance cycle, respectively. These times refer to durations within each cycle.

Date: 1 Dec 2022.
2010 *Mathematics Subject Classification.* Primary .
Key words and phrases. Stochastic models; Predictive Maintenance.

The symbols $\mathbb{P}$ and $\mathbb{E}$ represent the respective probability and expectation value belonging to $\mu : \mathcal{F} \to [0, 1]$. If not otherwise needed, we will drop the explicit dependence on a $\omega \in \Omega$ of these random variables in our notation.

Furthermore, let $\beta_i : \Omega \to \mathbb{R}_+$ be the i'th *breakdown time* of the machine, if no interruption of the production cycle is undertaken. This corresponds to the intended practice in reactive maintenance (RM). In preventive maintenance (PvM), on the other hand, this maximum period of activity is cut off at an intended τ_i when maintenance is initiated. We give expression to these two situations by introducing the random variable

$$\alpha_i := \tau_i \wedge \beta_i \tag{2.1}$$

(the minimum of τ_i and β_i) and call it the i'th *uninterrupted production time.* We distinguish the case when there is a timely maintenance at $\tau_i < \beta_i$ (i.e., $\alpha_i = \tau_i$) from when there is a premature breakdown ($\alpha_i = \beta_i$) in the i'th cycle before the planned maintenance takes place.

While preventive maintenance tries to choose the 'scheduled' period-length to be the fixed, non-random $\tau_i \in \mathbb{R}$ - sufficiently small to be smaller than $\beta_i(\omega)$, the specific maintenance strategy called Predictive Maintenance let's $\tau_i(\omega)$ depend on the specific situation, labelled by ω, in the i'th cycle.

Typically, $\tau_i(\omega)$ is the result of a remaining useful lifetime (RUL) prediction model $\mathcal{M} : \mathcal{X} \to \mathbb{R}_+$, taking some input variables $X_i \in \mathcal{X}$ at the beginning of the i'th cycle (see Section 3 for specific examples). The randomness of $\tau_i(\omega) = \mathcal{M}(X_i(\omega))$ then comes from the randomness of the input variables $X_i(\omega)$ of the RUL-prognosis (such as observed sensor values from a condition monitoring procedure).

To assess the influence of $\mathcal{M}$ (and its accuracy) on the profitability of using PdM instead of RM or PvM, we define the profit $P_i : \Omega \to \mathbb{R}$ incurred in the i'th cycle as the revenue obtained from uninterrupted production for a period of length α_i minus the repair-cost arising for repair of length ρ_i:

$$P_i(\omega) \;=\; g \cdot \alpha_i(\omega) \;\;-\;\; c \cdot \rho_i(\omega), \tag{2.2}$$

where $g, c \in \mathbb{R}_+$ are the gain rate during active production and c the rate of cost during repair (measured in money per unit time), respectively. Note that for reactive maintenance $\alpha_i = \beta_i$. For preventive maintenance $\alpha_i(\omega) = \tau \wedge \beta_i(\omega)$, where the use of ω in the notation is explicitly pointing out that τ is a fixed deterministic constant. In the general case of PdM, we have $\alpha_i = \tau_i(\omega) \wedge \beta_i(\omega)$.

For the complete i'th maintenance cycle's length $T_i : \Omega \to \mathbb{R}_+$ it is important to also consider an *idle time* $\widehat{\iota}_i : \Omega \to \mathbb{R}_+$, in which neither production nor maintenance takes place, adding to the total *downtime* $\delta_i : \Omega \to \mathbb{R}_+$, (i.e., $\delta_i(\omega) := \widehat{\iota}_i(\omega) + \rho_i(\omega)$), so that the cycle-length $T_i : \Omega \to \mathbb{R}_+$ consisting of production and downtime is

$$T_i(\omega) = \tau_i(\omega) \;+\; \widehat{\iota}_i(\omega) \;+\; \rho_i(\omega). \tag{2.3}$$

After n cycles, the total incurred profit is to be divided by sum of the n cycle-lengths giving an observed profit rate $p_n : \Omega \to \mathbb{R}_+$ defined by

$$p_n(\omega) \;:=\; \frac{\sum_{i=1}^{n} P_i(\omega)}{\sum_{i=1}^{n} T_i(\omega)}. \tag{2.4}$$

Under the condition of the random variables $\tau_i(\omega), \rho_i(\omega), \widehat{\iota}_i\rangle(\omega), \beta_i(\omega)$ connected to the i'th cycle and their derived variates $P_i(\omega), T_i(\omega)$ being identically distributed, the following proposition is immediate:

Proposition 1: *The limit* $\lim_{n\to\infty} p_n(\omega)$ *exists almost surely, is independent of* $\omega \in \Omega$*, and given by* $\mathbb{E}[P_1]/\mathbb{E}[T_1]$.

Proof: Let $\overline{P}_n(\omega) = \frac{1}{n}\sum_{i=1}^{n} P_i(\omega)$, and $\overline{T}_n(\omega) = \frac{1}{n}\sum_{i=1}^{n} T_i(\omega)$. By the law of large numbers the limits $\lim_{n\to\infty} \overline{P}_n = \mathbb{E}[P_1]$ and $\lim_{n\to\infty} \overline{T}_n = \mathbb{E}[T_1]$ exist almost surely. Since $p_n(\omega) = \overline{P}_n(\omega)/\overline{T}_n(\omega)$, and since the limit of $\overline{T}_n$ is positive the statement follows from the limit laws for fractions. □

We let $P = \lim_{n\to\infty} p_n(\omega)$ and call it the asymptotic profit rate. We now investigate different choices for g, c, α, and $\widehat{\iota}$ with respect to their effect on P.

2.2. Conditions for the Effective Use of Predictive Maintenance.

One of the key qualities of a PdM strategy is its ability to accurately predict the breakdown time β_i and schedule the end of the production run τ_i as close as possible before this breakdown occurs. We now show that this is immediately connected to the profitability of the strategy.

Consider the PdM case, in which $\tau_i(\omega)$ is random. Define the probability of timeliness Q as the probability that the scheduled production time is less than the breakdown time

$$Q_i \;=\; \mathbb{P}[\tau_i \;<\; \beta_i]. \tag{2.5}$$

Note that a large Q_i will *typically* corresponds to $\beta_i - \tau_i$ being large. It is a natural question how Q enters conditions which guarantee some advantage of using PdM instead of PvM or RM.

For the rest of the paper, we assume the process over the individual cycles to be stationary and ergodic, implying that the sequences of random variables $\tau_i, \rho_i, \beta_i, \alpha_i$, and ι_i ($\iota \in \mathbb{N}$) are identically and distributed and don't depend on the initial distribution ($x \mapsto \mathbb{P}[\tau_1 \;<\; x]$ etc.). Also, $Q = Q_i$ is independent of i.

We then note that due to this ergodic stationarity, $\mathbb{E}[\alpha_1] = \mathbb{E}[\tau_1]Q+(1-Q)\mathbb{E}[\beta_1]$, and we have

$$\mathbb{E}[\alpha_1] \;\;=\;\; \mathbb{E}[\beta_1] \;-\; \mathbb{E}[\beta_1 - \tau_1]\cdot Q. \tag{2.6}$$

The quantity $\gamma := \mathbb{E}[\beta_1 - \tau_1]\cdot Q$ also expresses how far the scheduled maintenance time deviates from the breakdown-time.

In the following discussion, we will be particularly interested in the effect of avoiding idle times through scheduled or predicted maintenance times. To show it most clearly, we will assume that for the PvM and PdM strategies, the idle time can be completely avoided, i.e., set to zero.

Proposition 2 The asymptotic profit rate P_R for reactive maintenance, and the asymptotic profit rate P_P for predictive or preventive maintenance without an idle time are, respectively, given by

$$P_R \;=\; \frac{g\mathbb{E}[\beta_1] \;-\; c\mathbb{E}[\rho_1]}{\mathbb{E}[\beta_1] \;+\; \mathbb{E}[\rho_1] \;+\; \mathbb{E}[\iota_1]} \text{ and } P_P \;=\; \frac{g\mathbb{E}[\beta_1] \;-\; c\mathbb{E}[\rho_1] - g\cdot\gamma}{\mathbb{E}[\beta_1] \;+\; \mathbb{E}[\rho_1] \;-\; \gamma}. \tag{2.7}$$

Remark: We consider preventive maintenance a special case of predictive maintenance with $\tau_i(\omega)$ being a degenerate (constant) random variable. This justifies the generalising symbol P_P.

Proof:(Prop. 2) From Prop. 1 it follows that the expectation values of P_1 (using (2.2)) and T_1 (using (2.3)) are used for the numerator and denominator, respectively. For P_P, (2.6) is used to express $\mathbb{E}[\alpha]$. □

Using these preliminaries, we are now able to formulate the conditions for the profitability of PdM or PvM over RM.

Theorem 1: The following conditions are equivalent:

(1) $P_R \; < \; P_P$
(2) $\mathbb{P}[\tau_1 < \beta_1] \cdot \mathbb{E}[\beta_1 - \tau_1] \; < \; \mathbb{E}[\iota_1] \cdot (g\mathbb{E}[\beta_1] \; - \; c\mathbb{E}[\rho_1]) \,/\, (g\mathbb{E}[\beta_1 + \iota_1] \; + \; c\mathbb{E}[\rho_1])$
(3) $\mathbb{P}[\tau_1 < \beta_1] \cdot \mathbb{E}[\beta_1 - \tau_1] \; < \; \mathbb{E}[\iota_1] \cdot \frac{P_R}{g \, - \, P_R}$

These results allow a thorough discussion of the various scenarios in different production environments. The most important two obersvatrions are these:

- It is only profitable to use PdM instad of RM if $\mathbb{E}[\iota] > 0$.
- Whenever $\mathbb{E}[\iota] > 0$, it is profitable to use PdM instead of RM if $\mathbb{P}[\tau_1 < \beta_1] \cdot \mathbb{E}[\beta_1 - \tau_1]$ is sufficiently small.

3. Conclusion and Outlook

This extended abstract gives an overview of the definitions and theoretical results obtained in our approach to model the profit gained from using PdM as opposed to RM and PvM. In the paper that will follow from this (and the presentation given at ENBIS 23), it will be shown how these results evaluate various prediction models $\mathcal{M}$ for the prognosis of the proper scheduled maintenance time τ.

4. Acknowledgement

The research reported in this paper has been funded by the European Interreg Austria-Czech Republic project "PredMAIn (ATCZ279)".

References

[1] Ashraf K.A.; Janardhanan Mukund Nilakantan; Patel Imran Adu-Amankwa, Kwaku; Attia. A predictive maintenance cost model for cnc smes in the era of industry 4.0. *The International Journal of Advanced Manufacturing Technology*, 2019.

[2] L.; Berenguer C.; Roussignol M. Grall, A.; Dieulle. Continuous-time predictive-maintenance scheduling for a deteriorating system. *IEEE Transactions on Reliability*, 51:141–150, 2002.

[3] S. Mohammad Hadi Hadavi. A heuristic model for risk and cost impacts of plant outage maintenance schedule. *Annals of Nuclear Energy*, 36:974–987, 2009.

[4] Hsu-Hua Lee. A cost/benefit model for investments in inventory and preventive maintenance in an imperfect production system. *Computers & Industrial Engineering*, 48(1):55–68, January 2005.

[5] Susan Lu; Yu-Chen Tu; Huitian Lu. Predictive condition-based maintenance for continuously deteriorating systems. *Quality and Reliability Engineering International*, 23:71–81, 2007.

SCCH - Software Competence Center Hagenberg GmbH - Softwarepark 32a, 4232 Hagenberg im Mühlkreis, AT

Email address: florian.sobieczky@scch.at