Processing new types of quality data: *analysis of partitioning metric space data*

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<table>
<thead>
<tr>
<th>2.1 Nominal data</th>
<th>2.2 Ordinal data</th>
<th>2.3 Ranked/prioritized data</th>
<th>2.4 Strings</th>
<th>2.5 Tree-structured data</th>
<th>2.6 Product or process distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Similarity/dissimilarity and closeness/distance</td>
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<td>3.3 Data aggregation or fusion and location measure</td>
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<td>3.5 Data partitioning</td>
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<td>3.6 Data clustering…</td>
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</tr>
</tbody>
</table>
Data/objects partition
Examples of distance metrics

- Hausdorff distance
- Euclidean distance
- Manhattan distance
- Minkowski distance
- Wasserstein distance
- Kendall tau distance
- Jensen-Shannon distance
- Geodesic distance
- Hellinger distance
- Hemming distance
Data/objects partition: degrees of connection (dc) split

\[ dc_{\text{total}} = dc_{\text{inter}} + dc_{\text{intra}} \]
Data/objects partition: *sum of distances (SD) split*

Sum of Distances: **Intra** \( (SD)_{\text{intra}} = d(A,B) + d(C,D) \)

Sum of Distances: **Inter** \( (SD)_{\text{inter}} = d(A,C) + d(A,D) + d(B,C) + d(B,D) \)

Sum of Distances: Total
\[ (SD)_{\text{total}} = d(A,B) + d(C,D) + d(A,C) + d(A,D) + d(B,C) + d(B,D) \]
Segregation power index - SP

\[ SP = \frac{MD_{\text{inter}}}{MD_{\text{intra}}} = \frac{SD_{\text{inter}} / dc_{\text{inter}}}{SD_{\text{intra}} / dc_{\text{intra}}} \]
Example 1: Distribution of quality costs proportions by the four main categories (Rosenfeld et.al, 2019)

<table>
<thead>
<tr>
<th>Cost Category</th>
<th>Company 1</th>
<th>Company 2</th>
<th>Company 3</th>
<th>Company 4</th>
<th>Company 5</th>
<th>Company 6</th>
<th>Company 7</th>
<th>Company 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prevention Costs</td>
<td>0.19</td>
<td>0.05</td>
<td>0.07</td>
<td>0.19</td>
<td>0.11</td>
<td>0.31</td>
<td>0.19</td>
<td>0.23</td>
</tr>
<tr>
<td>Appraisal Costs</td>
<td>0.22</td>
<td>0.15</td>
<td>0.13</td>
<td>0.35</td>
<td>0.47</td>
<td>0.12</td>
<td>0.32</td>
<td>0.23</td>
</tr>
<tr>
<td>Internal Failure Costs</td>
<td>0.14</td>
<td>0.28</td>
<td>0.18</td>
<td>0.27</td>
<td>0.19</td>
<td>0.14</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>External Failure Costs</td>
<td>0.45</td>
<td>0.52</td>
<td>0.62</td>
<td>0.19</td>
<td>0.23</td>
<td>0.43</td>
<td>0.22</td>
<td>0.27</td>
</tr>
</tbody>
</table>

![Bar Chart](chart.png)

Legend:
- Blue: Prevention Costs
- Orange: Appraisal Costs
- Gray: Internal Failure Costs
- Yellow: External Failure Costs
**Hellinger distances and SP calculation**

\[ H(P, Q) = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{4} \left( \sqrt{p_i} - \sqrt{q_i} \right)^2} \]

<table>
<thead>
<tr>
<th></th>
<th>Company 1</th>
<th>Company 2</th>
<th>Company 3</th>
<th>Company 4</th>
<th>Company 5</th>
<th>Company 6</th>
<th>Company 7</th>
<th>Company 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company 1</td>
<td>0.000</td>
<td>0.198</td>
<td>0.169</td>
<td>0.214</td>
<td>0.222</td>
<td>0.122</td>
<td>0.189</td>
<td>0.152</td>
</tr>
<tr>
<td>Company 2</td>
<td>0.198</td>
<td>0.000</td>
<td>0.094</td>
<td>0.290</td>
<td>0.290</td>
<td>0.265</td>
<td>0.265</td>
<td>0.240</td>
</tr>
<tr>
<td>Company 3</td>
<td>0.169</td>
<td>0.094</td>
<td>0.000</td>
<td>0.328</td>
<td>0.320</td>
<td>0.230</td>
<td>0.302</td>
<td>0.266</td>
</tr>
<tr>
<td>Company 4</td>
<td>0.214</td>
<td>0.290</td>
<td>0.328</td>
<td>0.000</td>
<td>0.120</td>
<td>0.269</td>
<td>0.030</td>
<td>0.104</td>
</tr>
<tr>
<td>Company 5</td>
<td>0.222</td>
<td>0.290</td>
<td>0.320</td>
<td>0.120</td>
<td>0.000</td>
<td>0.317</td>
<td>0.127</td>
<td>0.191</td>
</tr>
<tr>
<td>Company 6</td>
<td>0.122</td>
<td>0.265</td>
<td>0.230</td>
<td>0.269</td>
<td>0.317</td>
<td>0.000</td>
<td>0.244</td>
<td>0.178</td>
</tr>
<tr>
<td>Company 7</td>
<td>0.189</td>
<td>0.265</td>
<td>0.302</td>
<td>0.030</td>
<td>0.127</td>
<td>0.244</td>
<td>0.000</td>
<td>0.077</td>
</tr>
<tr>
<td>Company 8</td>
<td>0.152</td>
<td>0.240</td>
<td>0.266</td>
<td>0.104</td>
<td>0.191</td>
<td>0.178</td>
<td>0.077</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Summary Statistics**

- \( SD_{\text{intra}} = 1.727 \)  \( dc_{\text{intra}} = 12 \)  \( MD_{\text{intra}} = SD_{\text{intra}}/12 = 0.144 \)
- \( SD_{\text{inter}} = 4.082 \)  \( dc_{\text{inter}} = 16 \)  \( MD_{\text{inter}} = SD_{\text{inter}}/16 = 0.255 \)
- \( SD_{\text{total}} = 5.809 \)  \( dc_{\text{total}} = 28 \)  \( SP = 1.773 \)
Examples of 2 preference chains consisting of 4 alternatives

\[ C_1 : A_3 > A_4 > A_1 > A_2 \]
\[ C_2 : A_2 > A_1 > A_4 > A_3 \]
Example 2: Preference/prioritization chains

\[ C_1 : A_3 > A_4 > A_1 > A_2 \]
\[ C_2 : A_2 > A_1 > A_4 > A_3 \]

\[
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4 \\
A_1 & 0 & 1 & -1 & -1 \\
A_2 & -1 & 0 & -1 & -1 \\
A_3 & 1 & 1 & 0 & 1 \\
A_4 & 1 & 1 & -1 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4 \\
A_1 & 0 & -1 & 1 & 1 \\
A_2 & 1 & 0 & 1 & 1 \\
A_3 & -1 & -1 & 0 & -1 \\
A_4 & -1 & -1 & 1 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
(0 & 1 & -1 & -1 & -1 & 0 & -1 & -1 & 1 & 1 \ 0 & 1 & 1 & 1 & -1 & 0 & -1 & -1 & -1 & 1 & 0)
\end{array}
\]

\[
\begin{array}{cccc}
(0 & -1 & 1 & 1 & 1 & 0 & 1 & 1 & -1 & -1 \ 0 & -1 & -1 & -1 & 1 & 0)
\end{array}
\]
Distance metric based on cosine similarity

\[
sim(A, B) = \cos(\theta) = \frac{A \cdot B}{\|A\|\|B\|}
\]

\[
d(\vec{a}, \vec{b}) = \frac{\hat{\theta}_{\vec{a}, \vec{b}}}{\pi} = \arccos \left( \frac{\sum \forall_i a_i \cdot b_i}{\sqrt{\sum \forall_i a_i^2} \cdot \sqrt{\sum \forall_i b_i^2}} \right)
\]
Five experts/judges have prioritized alternatives with judges no. 2 and no. 3 being women, while judges no.1, no.4 and no.5 were men (Vanacore et.al, 2019)

<table>
<thead>
<tr>
<th>Judge</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.59</td>
<td>0.73</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>2</td>
<td>0.59</td>
<td>0</td>
<td>0.46</td>
<td>0.45</td>
<td>0.44</td>
</tr>
<tr>
<td>3</td>
<td>0.73</td>
<td>0.46</td>
<td>0</td>
<td>0.65</td>
<td>0.61</td>
</tr>
<tr>
<td>4</td>
<td>0.33</td>
<td>0.45</td>
<td>0.65</td>
<td>0</td>
<td>0.37</td>
</tr>
<tr>
<td>5</td>
<td>0.38</td>
<td>0.44</td>
<td>0.61</td>
<td>0.37</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
MD_{\text{intra}} = \left[ d_{2,3} + (d_{1,4} + d_{1,5} + d_{4,5}) \right]/4 = 0.385
\]

\[
MD_{\text{inter}} = \left[ (d_{2,1} + d_{2,4} + d_{2,5}) + (d_{3,1} + d_{3,4} + d_{3,5}) \right]/6 = 0.578
\]

\[
SP = MD_{\text{inter}} / MD_{\text{intra}} = 1.502
\]
$SP$ distribution under given null/ homogeneity hypothesis $H_0$

Does not depend on the location parameter of the origin data distribution

Does not depend on the scale parameter of the origin data distribution

Almost independent on shape parameter, especially for $SP > 1$

Depends only on the type of partition, i.e., vector $(n_1, n_2, \ldots, n_k, \ldots, n_m)$

(2,2)
How amount of data influence SP distribution under $H_0$

\[(10,10,\ldots,10)\]

\[n = 10\]

\[(n, n)\]

\[m = 2\]
Five experts/judges have prioritized alternatives with judges no. 2 and no. 3 being women, while judges no.1, no.4 and no.5 were men (Vanacore et.al, 2019)

<table>
<thead>
<tr>
<th></th>
<th>Judge j</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Judge i</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
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<tr>
<td></td>
<td>5</td>
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\[
MD_{\text{intra}} = [d_{2,3} + (d_{1,4} + d_{1,5} + d_{4,5})]/4 = 0.385
\]

\[
MD_{\text{inter}} = [(d_{2,1} + d_{2,4} + d_{2,5}) + (d_{3,1} + d_{3,4} + d_{3,5})]/6 = 0.578
\]

\[
SP = MD_{\text{inter}} / MD_{\text{intra}} = 1.502
\]
The null hypothesis:
“Gender equality and the absence of real preferences between alternatives”

$SP_{0.95} = 1.476$ and $p$ – value for calculated $SP$ = 1.502 equals 4.39%
**Modus operandi – 10 steps**

1. Decide on the population of objects under study (OUS).
2. Make assumption about the type of the expected distribution of these objects within a homogeneous population.
3. Choose distance metric suitable to this distribution
4. Decide on the criterion that, in your opinion, can influence (heterogeneity hypothesis) and which levels serve as the basis for dividing/separating objects into groups (partition).
5. Provide corresponding data partitioning/division
6. Calculate $SP$
7. Simulate $SP$ distribution under $H_0$ in accordance with partition vector $(n_1, n_2, \ldots, n_k, \ldots, n_m)$ and the chosen distance metric. Every cycle of simulation process includes:
   a. Random generation of $N$ data from a population of OUS (in accordance to step 1) characterized by the assumed distribution (in accordance to step 2).
   b. Distance matrix calculation (according to step 3)
   c. Partitioning these distances to *inter* and *intra* components in accordance to steps 4 and 5
   d. $SP$ calculation and back to a.
8. Determine the alpha risk - $\alpha$ of homogeneity hypothesis $H_0$ rejection.
9. Find $(1-\alpha)$ percentile of simulated $SP$ distribution, or alternatively, $p$ – value of calculated $SP$.
10. Make final conclusion about the expediency of the made partition and its discrimination/separation/segregation power by usual statistical methods according to previous step results.
Thank You for your attention!

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