Reliability Analytics: Cost-effective Condition-Based Maintenance Policy using Gamma Degradation Process

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ABSTRACT

Condition-based maintenance is an effective method to reduce unexpected failures as well as the operations and maintenance costs. This research discusses the condition-based maintenance policy with optimal inspection points under the gamma degradation process in order to improve the system reliability. A random effect parameter is used to account for population heterogeneities and its distribution is continuously updated at each inspection epoch. The observed degradation level along with the system age is utilized for making the optimal maintenance decision, and the structure of the optimal policy is examined.

INTRODUCTION

- Quantification of Reliability: survival probability
  - let T be the failure time of a product/unit
    $R(t) = P(T > t) = S(t) = 1 - F(t)$
  - non-increasing function of t

- CONDITION-BASED MAINTENANCE (CBM)
  - utilize modern sensor technology
  - System Health Management (SHM) through periodic inspections
  - maintenance actions based on the inspection of working conditions
  - proven effective in reducing unexpected failures with lower operational costs
  - outperform the traditional age-based and block-based maintenance policies

MAIN OBJECTIVES

- consider Gamma degradation process with random effects
- develop an **optimal condition-based maintenance/replacement policy** (MDP)
  - minimize the total operational costs
  - replacement decision based on the observed degradation level and the unit age
  - investigate the structural properties of the optimal policy: ***monotone control limit policy***
- determine an **optimal inspection interval**

GAMMA PROCESS

- monotone degradation path $\{Y_t,\ t \geq 0\}$
- independent increments:
  $\langle Y_{t_2} - Y_{t_1} \rangle$ and $\langle Y_{s_2} - Y_{s_1} \rangle$ are independent for $0 \leq s_1 < s_2 \leq t_1 < t_2$
- gamma distributed increments:
  $\langle Y_t - Y_s \rangle \sim Gamma(\alpha[\Lambda(t) - \Lambda(s)],\ \beta)$
  where $\Lambda(t)$ is a monotonically increasing time transformation function with $\Lambda(0) = 0$

- **Random Effect**
  - different units have different realizations of β
    $\beta \sim Inverse\ Gamma(\gamma,\ \lambda)$
    $$f(\beta;\ \gamma, \lambda) = \frac{\lambda^\gamma}{\Gamma(\gamma)} \beta^{-\gamma-1} e^{-\lambda/\beta}, \quad \beta > 0$$

- let $Y_j = Y_{t_j}$ be degradation levels observed at times t_j, $j = 1, 2, \ldots, n$
- let $\mathbf{Y}_n = (Y_1, Y_2, \ldots, Y_n)$ and $\Lambda_j = \Lambda(t_j)$

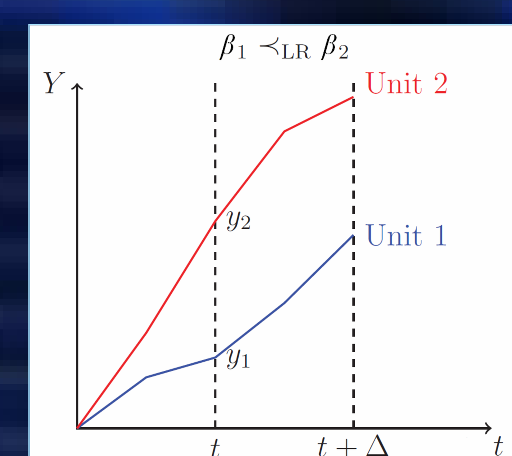
- $\langle \beta | \mathbf{Y}_n \rangle \sim Inverse\ Gamma(\alpha\Lambda_n + \gamma,\ Y_n + \lambda)$
  - updated posterior depends only on Y_n and Λ_n

- $\left\langle \frac{Y_{n+1} - Y_n}{Y_{n+1} + \lambda} \middle| \mathbf{Y}_n \right\rangle \sim Beta(\alpha[\Lambda_{n+1} - \Lambda_n],\ \alpha\Lambda_n + \gamma)$
  - *Proof:* $f(y_{n+1} | \mathbf{y}_n) = \int_0^\infty f(y_{n+1} - y_n | \beta)\ f(\beta | \mathbf{y}_n)\ d\beta$

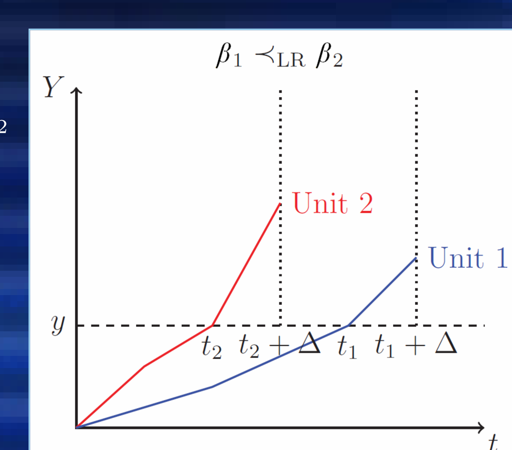
- $\langle Y_{n+1} | \mathbf{Y}_n \rangle$ depends only on Y_n
  - **Markov property**

STOCHASTIC PROPERTIES

- **Lemma 1.**
  $\langle Y_{t+\Delta} | Y_t \rangle$ is stochastically non-decreasing in Y_t
  viz., $\langle Y_{t+\Delta} | Y_t = y_1 \rangle \prec \langle Y_{t+\Delta} | Y_t = y_2 \rangle$ given $y_1 < y_2$



- **Lemma 2.**
  $\langle Y_{t+\Delta} | Y_t \rangle$ is stochastically non-increasing in t
  viz., $\langle Y_{t_1+\Delta_1} | Y_{t_1} = y \rangle \prec \langle Y_{t_2+\Delta_2} | Y_{t_2} = y \rangle$ given $t_1 > t_2$ and $\Lambda(t_1 + \Delta_1) - \Lambda(t_1) \leq \Lambda(t_2 + \Delta_2) - \Lambda(t_2)$

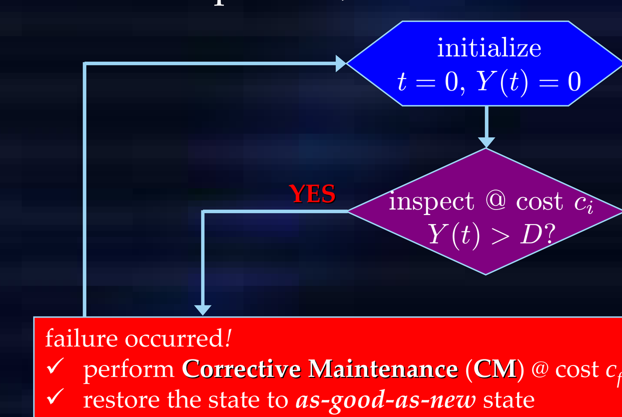


CBM PARAMETERS

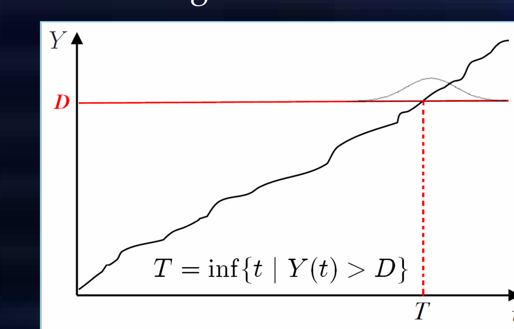
- let
  - δ = inspection interval
  - c_i = inspection cost
  - c_f = corrective replacement cost
  - c_p = preventive maintenance cost ($c_p < c_f$)
  - c_d = downtime cost per unit time
  - $\exp(-rt)$ = discounting factor

CBM PROCEDURE

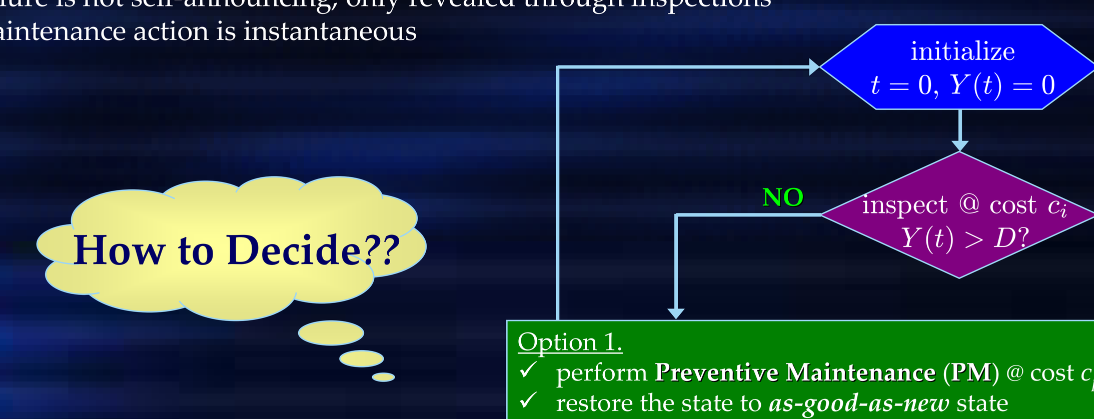
- periodic inspection with $\Lambda(t) = t$ for illustration
- at each inspection, make decision

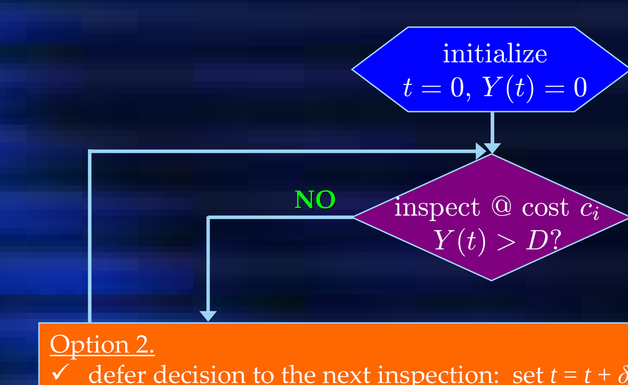


D = degradation threshold



- failure is not self-announcing, only revealed through inspections
- maintenance action is instantaneous





- failure may occur @ unknown failure time
  ➔ **downtime cost** incurs due to loss of efficiency or quality if operating in the failure state

MARKOV DECISION PROCESS

- $(\tau_k,\ Y_{\tau_k})$ forms discrete-time continuous-state Markov chain with age-dependent transition probability
- at each state, take one action from **CM**, **PM**, NULL

- **Value Function**
  - minimum total discounted cost, starting from the state (u,v) on the infinite time horizon
  - satisfy the Bellman equation; Puterman (2009)

$$V_\delta(u,v) = \begin{cases} \min\left\{e^{-r\delta} U_\delta(u,v) + W_\delta(u,v),\ c_p + V_\delta(0,0)\right\}, & v \leq D; \\ c_f + V_\delta(0,0), & v > D \end{cases}$$

  for equi-spaced inspection time $u = 0, \delta, 2\delta, \ldots$ and degradation level $v > 0$, where

  $U_\delta(u,v) = E\left[V_\delta(u+\delta, Y_{u+\delta}) \mid \tau_k = u, Y_{\tau_k} = v\right]$ is the expected value with one period transition from the current state

  $W_\delta(u,v) = E\left[\rho(T_k) \mid \tau_k = u, Y_{\tau_k} = v\right]$ is the expected downtime cost based on the current state

- expected total maintenance cost $V_\delta(0,0)$
- total discounted inspection cost $S(\delta) = \sum_{k=0}^{\infty} c_i e^{-rk\delta} = c_i\left(1 - e^{-r\delta}\right)^{-1}$

➔ minimize the ***total operational costs*** $V_\delta(0,0) + S(\delta)$
- find the optimal inspection interval δ*
- find the corresponding maintenance policy

STRUCTURE of OPTIMAL POLICY

- **Lemma 3.**
  $U_\delta(u,v)$ is non-decreasing in v and non-increasing in u
  $W_\delta(u,v)$ is non-decreasing in v and non-increasing in u

Value Iteration Algorithm

```
begin
    Initialize: V^0(τ_k, Y_τk) ⟵ 0;
    Compute: P(Y_τk+1 | Y_τk, τ_k) and W(τ_k, Y_τk) for all τ_k, Y_τk;
    while |V^s(τ_k, Y_τk) − V^{s−1}(τ_k, Y_τk)| < ϵ do
        if Y_τk > D then
            V^{s+1}(τ_k, Y_τk) = c_f + V^s(0, 0);
        else
            V^{s+1}(τ_k, Y_τk) = e^{−rδ} ∑ V^s(τ_k+1, Y_τk+1) ·
            P(Y_τk+1 | Y_τk, τ_k) + W(τ_k, Y_τk);
            if V^{s+1}(τ_k, Y_τk) > c_p + V^s(0, 0) then
                V^{s+1}(τ_k, Y_τk) = c_p + V^s(0, 0);
            end
        end
    end
end
```

- **Theorem 1.**
  Given δ, the optimal maintenance policy that minimizes $V(0,0)$ is a monotone control limit policy. $\exists$ a non-decreasing sequence $\{\xi_k\}$ such that the optimal action at state $(k\delta, y)$ is PM if $y > \xi_k$, and NULL otherwise

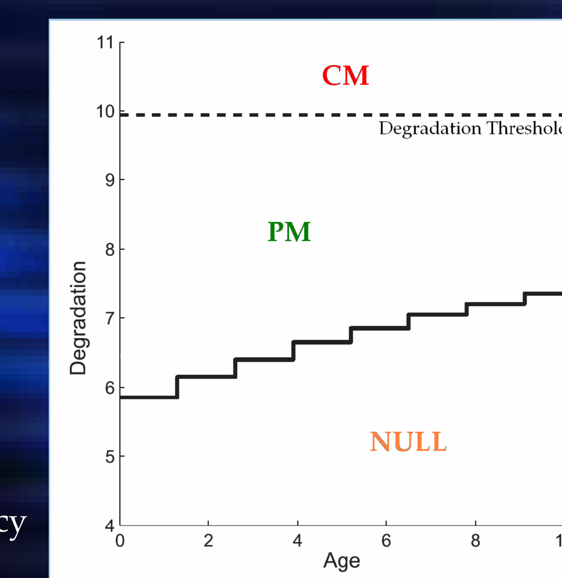


Figure 1. Optimal maintenance policy with the inspection interval δ = 1.3

OPTIMAL INSPECTION

- $\lim_{\delta \to \infty} V_\delta(0,0) + S(\delta) = E\left[c_d e^{-rT}/r \mid Y_0 = 0\right]$
  - $V_\delta(0,0)$ increases in δ to a constant as the downtime cost dominates
  - $S(\delta)$ decreases in δ to 0

- $\delta^* = \arg\min_\delta \mathrm{V}_\delta(0,0) + S(\delta)$
  - minimize total cost
  - robust as flat cost near δ*

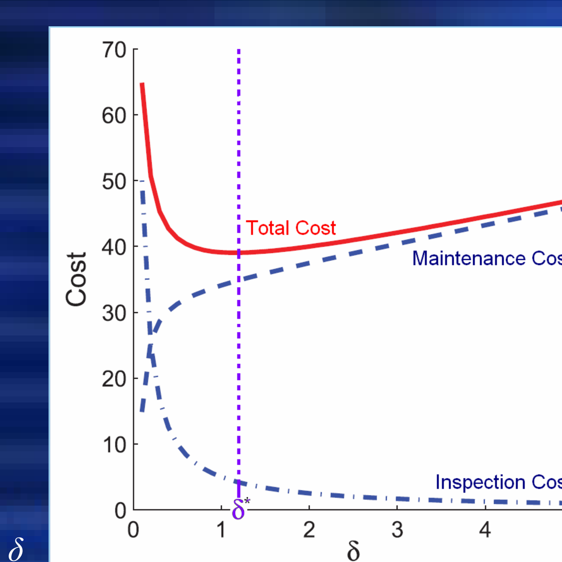


Figure 2. Cost functions at different inspection interval δ

REFERENCES

- Bloch-Mercier, S. (2002). A preventive maintenance policy with sequential checking procedure for a Markov deteriorating system. *European Journal of Operational Research*, **142:** 548–576.
- Chen, N., Ye, Z.S., Xiang, Y., and Zhang, L. (2015). Condition-based maintenance using the inverse Gaussian degradation model. *European Journal of Operational Research*, **243:** 190–199.
- Elwany, A.H., Gebraeel, N.Z., and Maillart, L.M. (2011). Structured replacement policies for components with complex degradation processes and dedicated sensors. *Operations Research*, **59:** 684–695.
- Guo, C., Wang, W., Guo, B., and Si, X. (2013). A maintenance optimization model for mission-oriented systems based on Wiener degradation. *Reliability Engineering and System Safety*, **111:** 183–194.
- Hong, Y., Ye, Z.S., and Xie, Y. (2013). How do heterogeneities in operating environments affect field failures predictions and test planning? *The Annals of Applied Statistics*, **7:** 1837–2457.
- Lawless, J. and Crowder, M. (2004). Covariates and random effects in a gamma process model with application to degradation and failure. *Lifetime Data Analysis*, **10:** 213–227.
- Puterman, M.L. (2009). *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. Hoboken, NJ: Wiley.
- Shaked, M. and Shanthikumar, J.G. (2007). *Stochastic Orders*. NY: Springer.
- Wang, X. (2010). Wiener processes with random effects for degradation data. *Journal of Multivariate Analysis*, **101:** 340–351.
- Wu, S. and Zuo, M.J. (2010). Linear and non-linear preventive maintenance models. *IEEE Transactions on Reliability*, **59:** 242–249.