Application of Simplicial Functional Data Analysis to Statistical Process Control in Additive Manufacturing

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Introduction

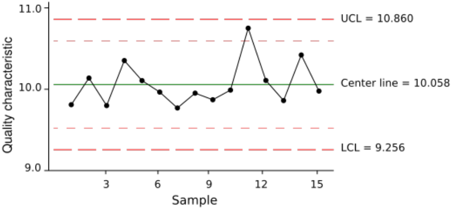
Motivating examples: control of complex parts

Metal Additive Manufacturing productions

a) lightweight bracket for space applications, b) topologically optimized space antenna support and c) rocket engine demonstrator

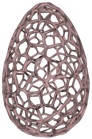
Object characterization: from simple to complex shapes

Clear quality features

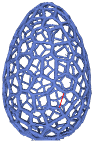


For simple parts as screw and bolts, quality features are straightforward to identify, and uni- or multi-variate control charts can be built

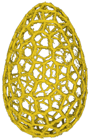
What quality features?



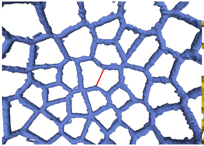
(a) Prototype



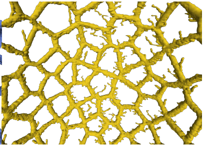
(b) Defective-MS



(c) Defective-EM



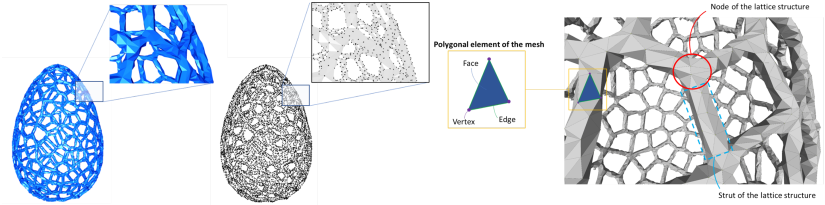
(d) Defective-MS



(e) Defective-EM

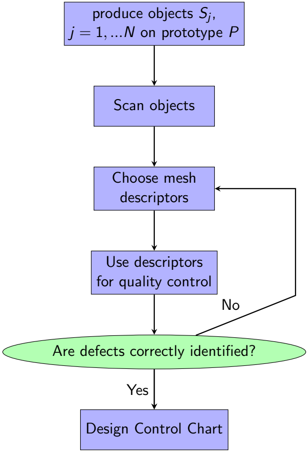
The egg shells were manufactured via AM at Dipartimento di Meccanica, Politecnico di Milano

Mesh and Point Cloud data



Mesh and point cloud data are obtained by X-ray Computed Tomography on the manufactured shapes

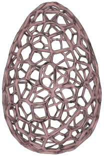
General Control Framework



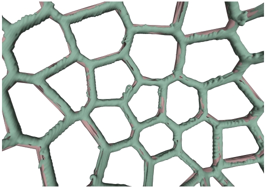
Modeling defects and geometric deviations

Prototype vs manufactured part

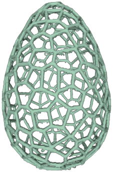
Mesh and Point Cloud data



(a) NominalModel mesh



(b) Geometrical mismatch after ICP algorithm



(c) Printed mesh

How should we capture all information about geometrical deviations?

Hausdorff distance

With P we denote the prototype point cloud, with S_j the mesh of a scanned object.

Natural Metric between sets

$P, S_j \subset \mathbb{R}^3$ — geometrical setting

$d_{S_j}(p) := \min_{s \in S} d(p, s) \, \forall p \in P$ — deviation map from P to S_j

$d_P^j(s) := \min_{p \in P} d(s, p) \, \forall s \in S$ — deviation map from S_j to P

$d_H(P, S) := \max\{\max_{p \in P} d_{S_j}(p), \max_{s \in S} d_P^j(s)\}$ — Hausdorff distance

Metric between subsets of a metric space, naturally induced by the metric space itself.

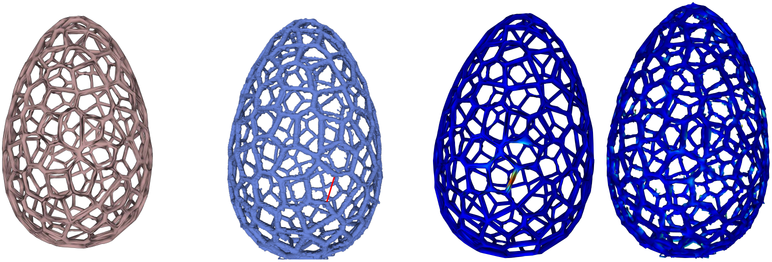
Geometric interpretation

Defect characterization

Since $d_H(P, S_j) = 0 \iff P = S_j$, the couple of maps (d_{S_j}, d_P^j) fully characterizes the geometrical differences between the point clouds.

- The two maps generally carry different and complementary information
- In previous works, where simple objects and defects were considered, only one deviation map is analyzed
- The deviation maps are spatial functions with a different 3D domain. Moreover, the d_P^j have different domains and cannot be directly compared.

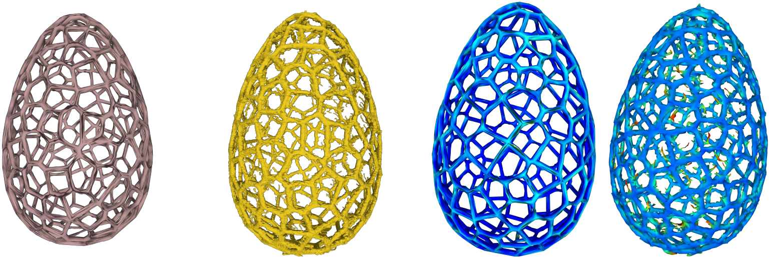
Example-complementarity of the maps



(a) Prototype **(b)** Defective-MS **(c)** map d_{S_j} **(d)** map d_P^j

- $d_P^j : S_j \to \mathbb{R}$, $d_P^j(s) := \min_{p \in P} d(s, p)$ cannot see the defect in this case (no points associated to high values of distance)
- $d_{S_j} : P \to \mathbb{R}$, $d_{S_j}(p) := \min_{s \in S} d(p, s)$, can!

Example-complementarity of the maps



(a) Prototype **(b)** Defective-EM **(c)** map d_{S_j} **(d)** map d_P^j

- Here the situation is reversed!

From Distance Maps to Densities

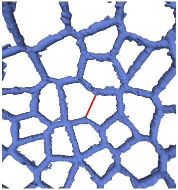
Summarizing maps for proper comparison

We abandon any spatial reference

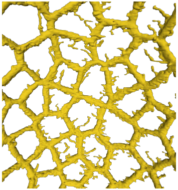
- $d_{S_j} \rightarrow f_{S_j}$, density of distances of points of P from S_j
- $d_P^j \rightarrow f_P^j$, density of distances of points of S_j from P
- Two N-dimensional datasets, f_S and f_P, with a precise geometric interpretation

From Distance Maps to Densities

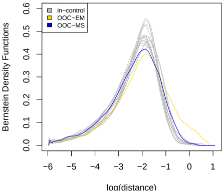
From N objects to 2N densities



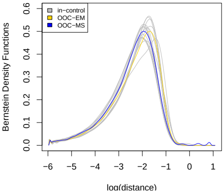
(a) OOC-MS



(b) OOC-EM



(c) f_P



(d) f_S

The initial dataset of 16 trabecular egg shells has been represented by two datasets of probability densities. Densities are estimated via Bernstein polynomials

Why densities?

- Natural extension of what is done in previous works (study of moments, QQ-plots)
- The mathematical theory is solid enough to extend SPC tools as control charts

B^2 Hilbert space of densities

- $B^2(a,b) := \{f : [a,b] \to \mathbb{R}\ measurable\ s.t.\ f > 0\,, \log f \in L^2(a,b)\}$, $f = g \iff f = cg$
- $f + g := fg$, $\alpha \cdot f := f^{\alpha}$
- $\langle f, g \rangle_{B^2} = \frac{1}{2(b-a)} \iint \log \frac{f(x)}{f(y)} \log \frac{g(x)}{g(y)}\, dxdy$

Control Framework

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Profile Monitoring of density functions

Summarizing a dataset of densities

For control, we need to build appropriate statistics:

- PCA is consistently extended to Hilbert spaces and thus to B^2 (SFPCA, Hron et al., 2016)
- Standard PCA-based control can then be applied

Profile Monitoring of density functions

Computing scores

Let $(H, +, \langle\cdot,\cdot\rangle)$ be Hilbert, $\{X_i\}_{i=1,\ldots N} \subset H$ a dataset with zero mean and sample covariance Σ, that is $\Sigma u = \frac{1}{N}\sum_i \langle X_i, u\rangle X_i \quad \forall u$. Let $(\lambda_j, \zeta_j)_{j=1,\ldots N}$ be the spectral decomposition of Σ, and $z_{ij} = \langle X_i, \zeta_j\rangle$ the scores. Fix $K \in \{1, \ldots, N-1\}$ suitably.

- $T_i^2 = \sum_{j=1}^{K} \frac{z_{ij}^2}{\lambda_j}$ measures the distance between the mean and the reconstruction of X_i on the K-th principal subspace $span(\zeta_1, \ldots \zeta_K)$, taking into account the data variability
- $Q = \sum_{j=K+1}^{N} z_{ij}^2$ measures the Euclidean distance between the mean and the part of X_i outside the first $K-$th principal subspace.
- T^2 and Q are uncorrelated and can be used for control, as in classical PCA-based control charts

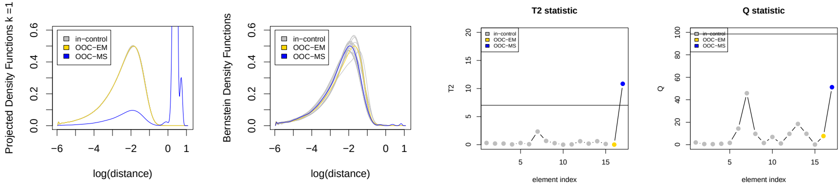
Control charts construction on real data

Two couple of charts

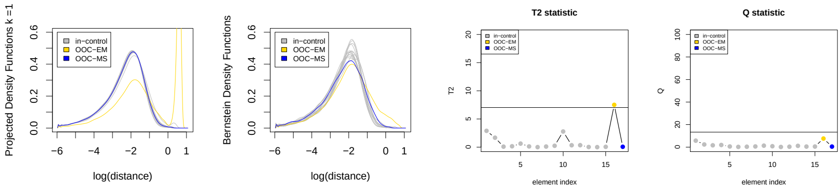
- T^2 and Q charts for the f_{S_j}, $j = 1, ...N$ densities.
- T^2 and Q charts for the f_P^j, $j = 1, ...N$ densities.
- Control limits can be set empirically or via approximated results
- An element is out of control if any of the four chart raises an alarm

Control charts construction on real data

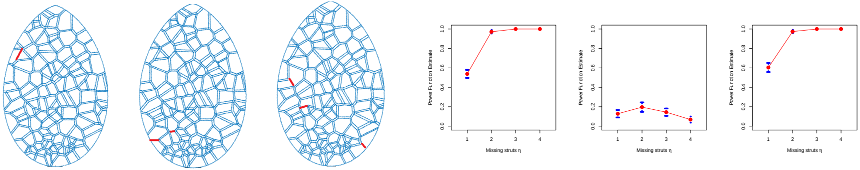
Control charts on f_{S_j} densities



Control charts on f_P^j densities



Simulations for power estimation



(a) 1 strut missing

(b) 2 strut missing

(c) 3 strut missing

(d) f_{S_j} analysis

(e) f_P^j analysis

(f) Overall power

Other scenarios were explored, with very satisfactory results

Conclusion

Final Remarks

A general strategy

The choice of the shape descriptors, in conjunction with the theory of Hilbert spaces of densities allow us to:

- Build a general framework for SPC on dataset of scanned objects, regardless of their complexity or topological richness
- Summarize the “defective” or “conformal” status of an object on the basis of simple statistics
- Design extensive simulation studies
- Detect both widespread and very local defectiveness sources

Main References

1. **Scimone, R., Taormina, T., Colosimo, B.M., Grasso, M.L., Menafoglio, A., Secchi, P. (2021): Statistical modeling and monitoring of geometrical deviations in complex shapes with application to Additive Manufacturing, MOX report**, at `https://www.mate.polimi.it/biblioteca/add/qmox/23-2021.pdf`.
2. Menafoglio, A., M. Grasso, P. Secchi, and B. Colosimo (2018) : Profile Monitoring of Probability Density Functions via Simplicial Functional PCA With Application to Image Data". Technometrics 60, pp. 497–510.
3. Egozcue, J. J., L. Diaz–Barrero, J., and Pawlowsky-Glahn, V. (2006). Hilbert space of probability density functions based on Aitchison geometry. Acta Mathematica Sinica, English Series, 22, pp. 1175–1182.
4. Hron, K., Menafoglio, A., Templ, M., Hruzova, K., and Filzmoser, P. (2014). Simplicial principal component analysis for density functions in Bayes spaces. Computational Statistics & Data Analysis 94, pp. 330–350.
5. Wells, L. J., Megahed, F. M., Niziolek, C. B., Camelio, J. A., and Woodall, W. H. (2013). Statistical process monitoring approach for high-density point clouds. Journal of Intelligent Manufacturing 24, pp. 1267–1279.