Video/Image Statistical Process Monitoring in Additive Manufacturing via Partial First Order Stochastic Dominance

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- With highly customized products, we lack the ability of establishing a traditional phase I "reference" behavior as we have a dynamically changing setup. In-situ and online process monitoring by means of video/image data shall then be combined with a novel way of designing the control charts used to automatically signal any departure from a natural, but dynamically changing, behavior.

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- With 0 (255) being the absolute black (white), we will have that small pixel values ("dark pixels") will refer to background, while large pixel values ("bright pixels") will refer to foreground.

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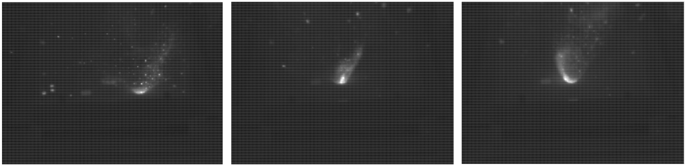

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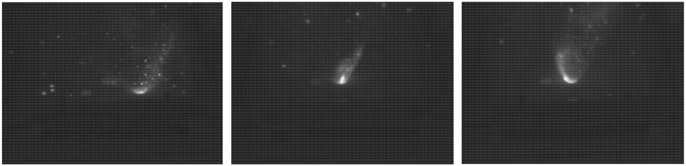
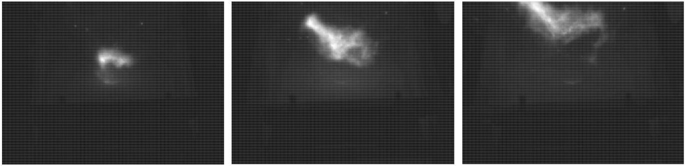


c) Examples of video frames with OOC exploding plume emissions



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- In this spirit several authors attempted to solve the problem. Namely:
  - Wang and Tsung (2005) used the idea of testing with QQ-plots the conformance of the new incoming frames against IC frames.
  - Menafoglio et. all (2018) suggested profile monitoring of the empirical pdf in a Bayes-Hilbert space, to test agreement of an incoming frame against IC patterns, established in phase I exercise.

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  - Menafoglio et. all (2018) suggested profile monitoring of the empirical pdf in a Bayes-Hilbert space, to test agreement of an incoming frame against IC patterns, established in phase I exercise.
- Our proposal is to remain in the area of non-parametrics utilizing first order stochastic dominance (FOSD) properties.

Stochastic Dominance

First Order Stochastic Dominance

In the univariate setting, we say that a random variable X with cdf $F_X(\cdot)$ will be first order stochastic dominant over the random variable Y with cdf $F_Y(\cdot)$, denoted as $Y \overset{sd}{\leq} X$ if:

$$F_X(t) \leq F_Y(t) \;\; \forall t \in \mathbb{R} \quad \text{and } \exists t^* \in \mathbb{R} \quad \text{such that:} \quad F_X(t^*) < F_Y(t^*)$$

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- if $h(\cdot)$ is any bounded increasing function we have that:

$$Y \overset{sd}{\leq} X \Rightarrow E[h(Y)] < E[h(X)]$$

Stochastic Dominance

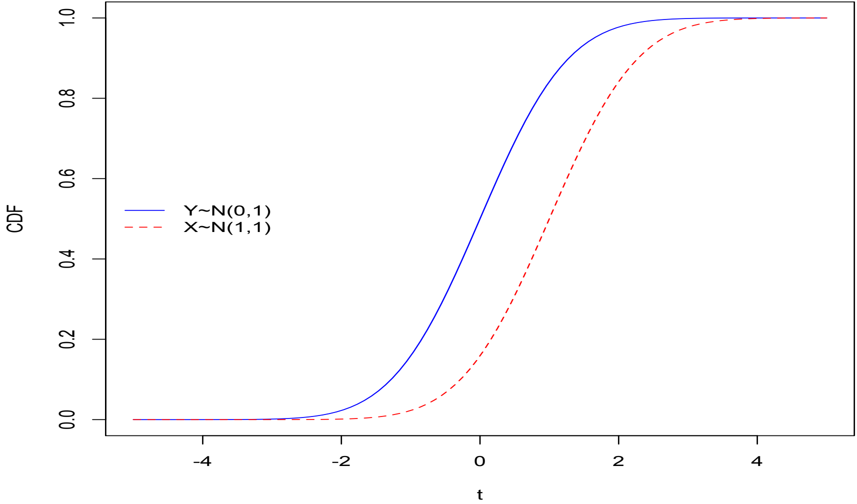

Figure: An example of first order stochastic dominance (X dominates Y)

Stochastic Dominance

Partial First Order Stochastic Dominance

We call partial stochastic dominance, when the concept of stochastic dominance is restricted over a predetermined set A. We say $Y \overset{psd_A}{\leq} X$ if:

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- We will make use of the empirical cumulative distribution function (ecdf) of all pixel values of each frame, to summarize the ongoing "action" of a frame.
- Using a phase I calibration to establish what is the IC ecdf behavior and taking into account that OOC is considered as "excessive action" compared to IC, we will utilize the use of partial first order stochastic dominance (FOSD) properties to test (online during phase II), when we move from the IC to the OOC state.

ecdfs of extreme images

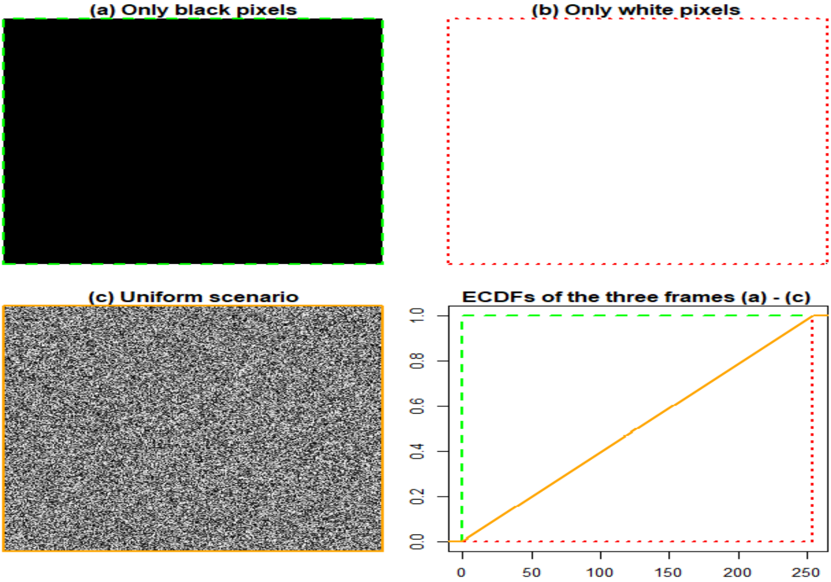

Image based SPC/M

- Using ecdfs we will attempt to slightly generalize the concept of testing against an IC "prototype". The idea of OOC performance in the image based SPC/M that we consider in this study is reflected as bigger "action" space (area) compared to what was established during the IC phase I.

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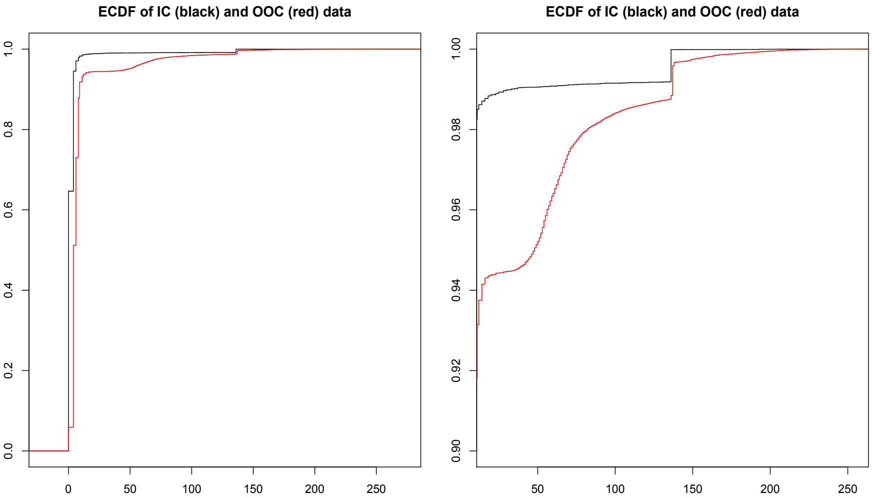
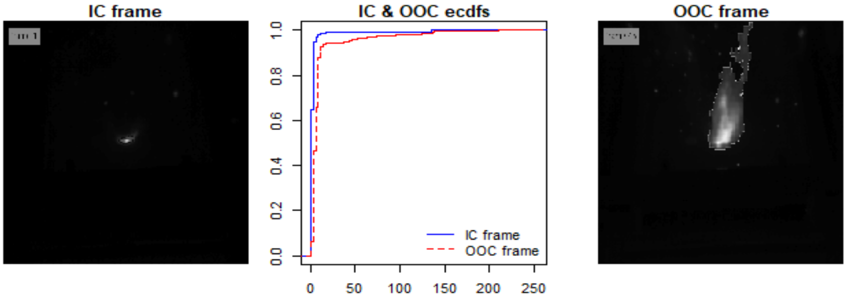
- In other words we expect in OOC situations to have more pixels with high values compared to the IC video sequence.
- Generally speaking in the discrete pmf over $\{0, 1, 2, \ldots, 254, 255\}$ as we move from an IC to an OOC frame "chunks" of probability mass will travel from the smaller to the bigger values forcing the ecdf to be moved to the right.

IC versus OOC ecdfs



IC and OOC ecdfs



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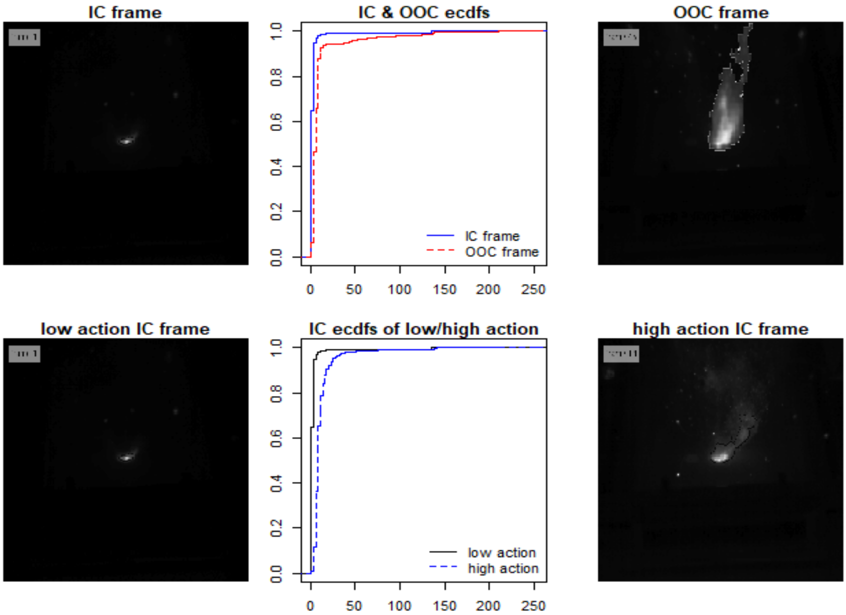

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  - "Foreground" area, where the action is taking place (related to the high pixel values) and
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  - "Background" area, where no significant action is observed, i.e. we can think of it as the "quite" region of the frame (relates to the small pixel values of the support set).
- The "foreground" region can reflect the IC or OOC status of a frame. Namely, if we will identify the "foreground subset" of the support space, we expect an OOC ecdf to be partially stochastically greater from the typical IC ecdf over this region. Making use of the partial stochastic dominance properties we will expect to have smaller area under the ecdf for an OOC frame, compared to the respective area of an IC frame, over the "foreground subspace".

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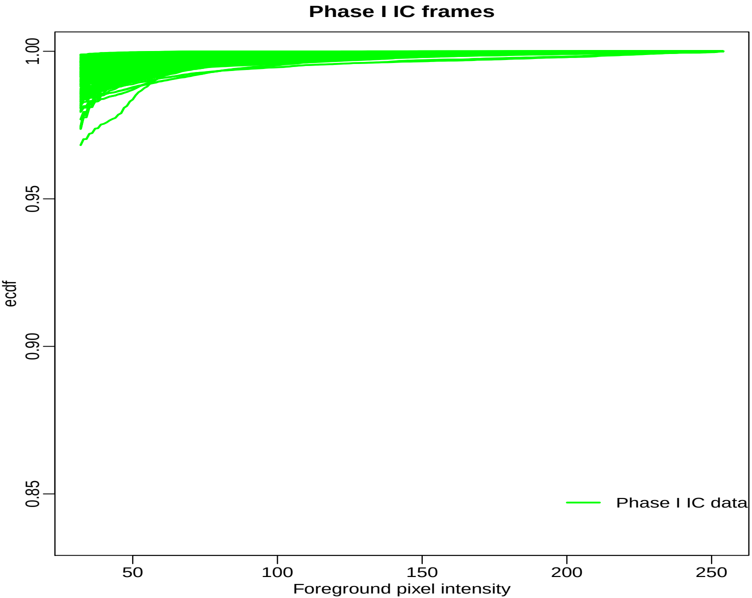

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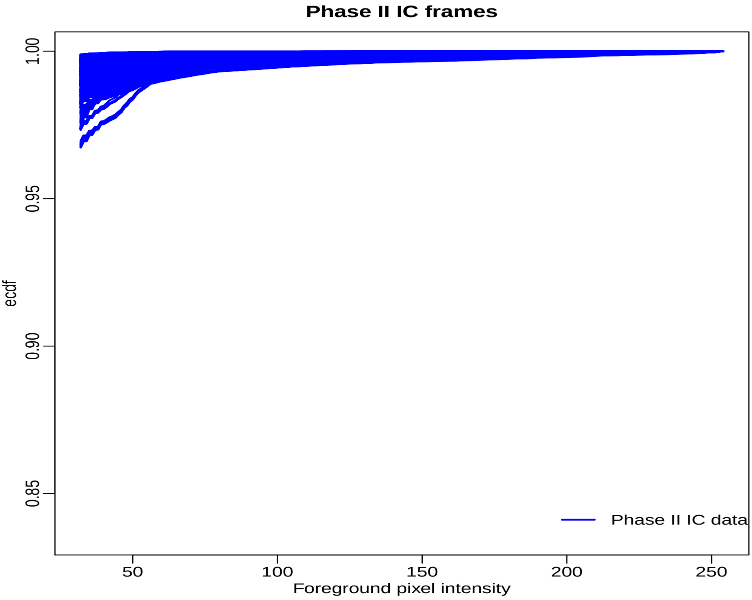

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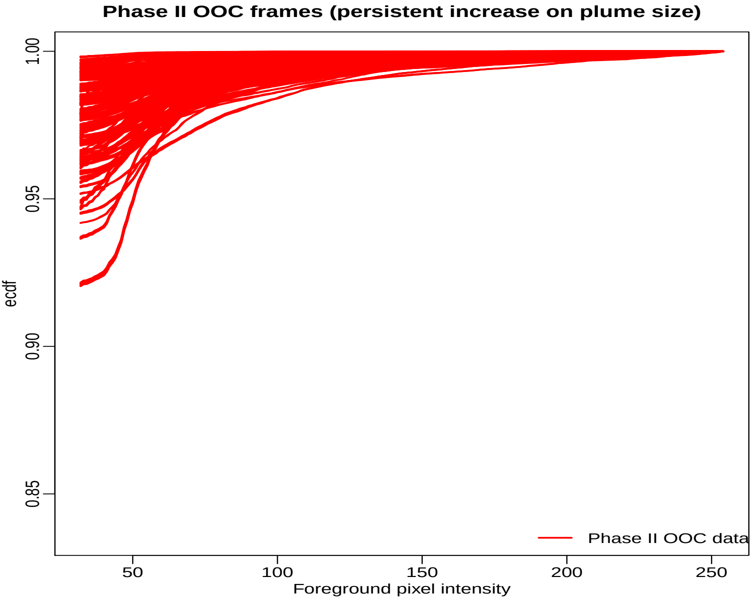

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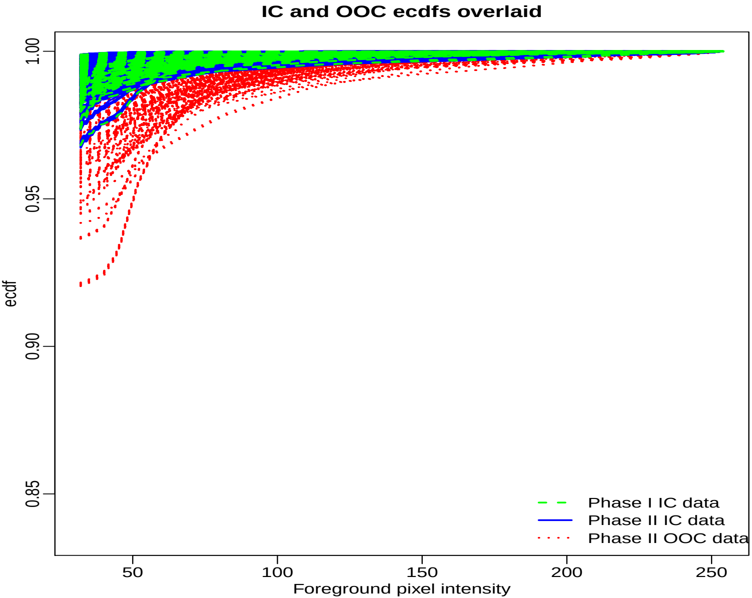

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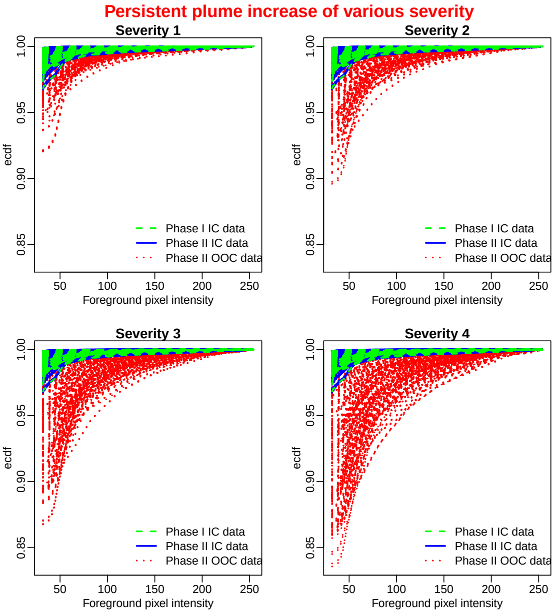

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- Plot these thresholds in a control type of chart, where on vertical axis we have the $Area_i$ and on the horizontal axis the frame number.
- Move to online phase II monitoring: for each new frame, derive its ecdf and the respective area under the ecdf over S_f and plot this area on the control chart. A point below the lower limit threshold will indicate rejection of the IC scenario.

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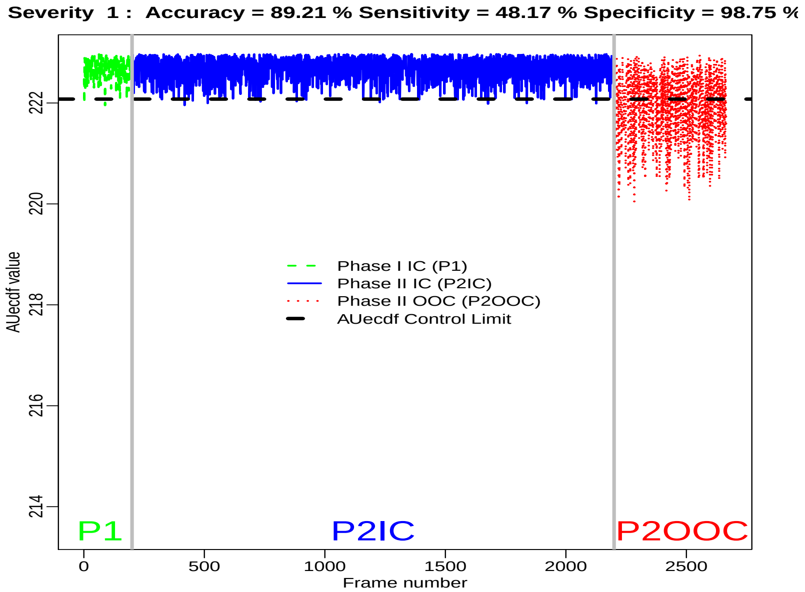

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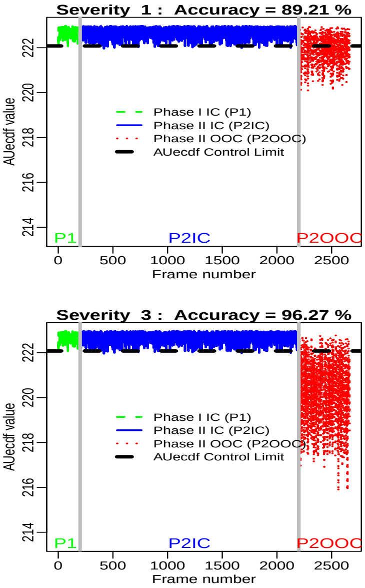

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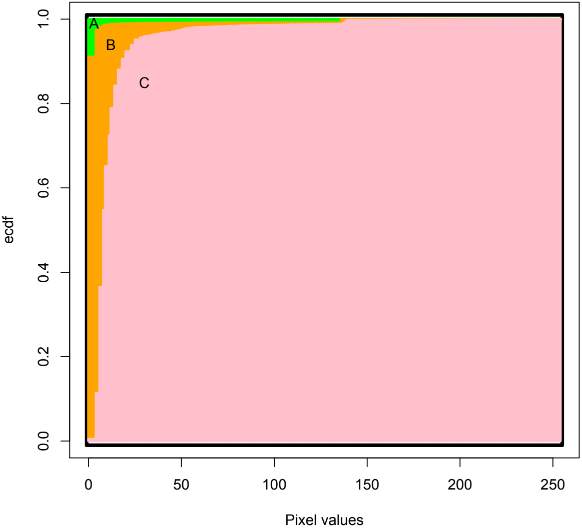

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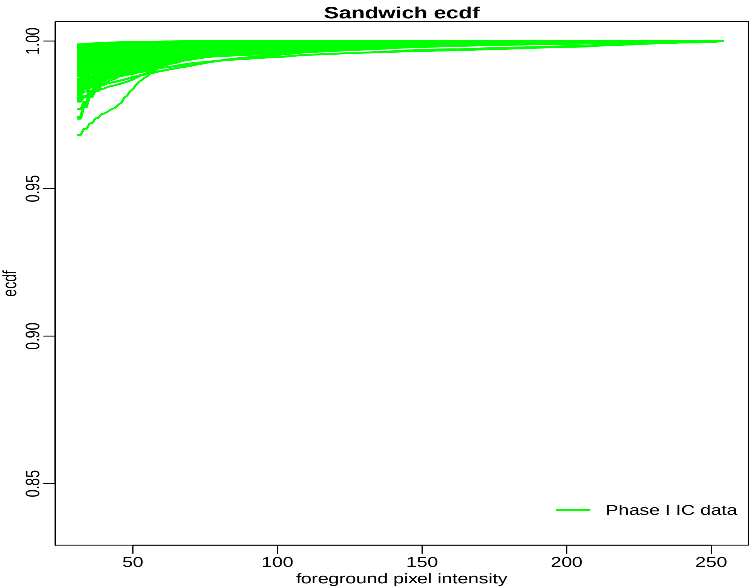

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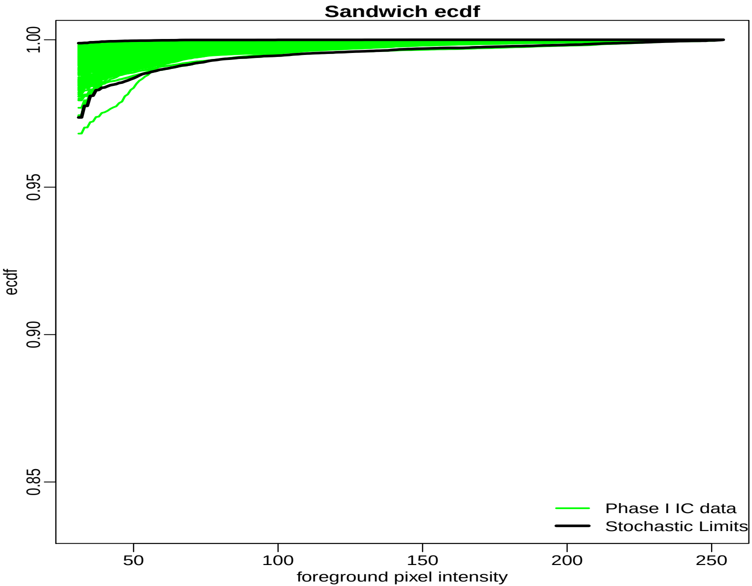

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- The new proposal outperforms the profile monitoring of pixel intensities' Q-Q plots alternative.
- The suggested method can be generalized for other than additive manufacturing performance (e.g. affective computing).

The End

Thank you!