Predictive Control Charts (PCC):
Bayesian Approach in Online Monitoring of Short Runs

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- PCC will utilize the available prior information (or adopt an objective Bayesian approach in case of prior ignorance) and it will be formed as a sequentially updated region based on the predictive distribution.
- PCC will be introduced in a general form, allowing to handle data of any univariate (discrete or continuous) distribution, as long as this distribution is a member of the k-Parameter Regular Exponential Family (k-PREF).

PCC: statistical modeling

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- Our main interest is in detecting in an online fashion and without employing a phase I exercise, the presence of large transient shifts (outliers) on the unknown parameter(s) $\boldsymbol{\theta}$.

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where $\boldsymbol{Y} = (y_1, \ldots, y_{n_0})$ are the historical data, $0 \leq \alpha_0 \leq 1$ is a scalar parameter, $\pi_0(\boldsymbol{\theta}|\boldsymbol{\tau})$ is the **initial prior** for the unknown parameter(s) and $\boldsymbol{\tau}$ is the vector of the initial prior hyperparameters.

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- From an objective Bayesian point of view we can adopt for $\pi_0(\cdot)$ a weakly informative or even non-informative initial prior, such as flat/Jeffreys/reference/... prior.

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while the predictive distribution of the single future observable X_{n+1} will be:

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- Precisely, we will determine an IC region, R_{n+1}, where the future observable (X_{n+1}) will most likely be, as long as the process is stable (i.e. no changes occurred).

PCC: Highest Predictive Density/Mass (HPrD/M)

- Assume the set R^c which contains the values of the predictive density (or mass) function, which are greater than a threshold c, i.e.:

  $$R^c = \{x_{n+1} : f(x_{n+1} | \boldsymbol{X}, \boldsymbol{Y}, \alpha_0, \boldsymbol{\tau}) \geq c\}$$

  The HPrD/M region will be given by minimizing the absolute difference of a highest predictive probability from a significance level $1 - \alpha$, for all the possible values of c. Specifically:

  $$R_{n+1} = \min_{R^c} \left| \int_{R^c} f(x_{n+1} | \boldsymbol{X}, \boldsymbol{Y}, \alpha_0, \boldsymbol{\tau}) - (1 - \alpha) \right|,$$

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- R_{n+1} will be the shortest region with the smallest absolute difference from the probability $1 - \alpha$.

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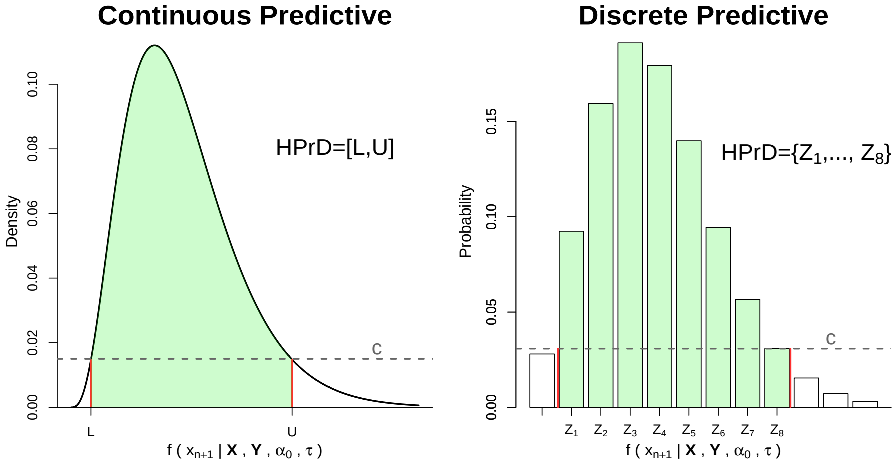

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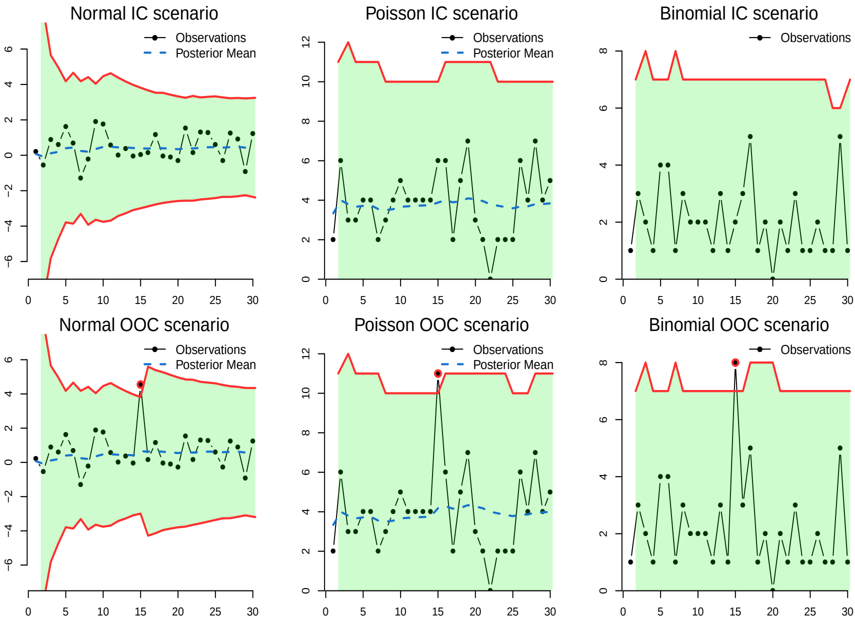
  2. If N is either unknown in advance or it is too large, then we suggest to derive α using the metric of IC Average Run Length (ARL_0):

  $$\alpha \approx \frac{1}{ARL_0}$$

PCC: Illustration and Decision Making



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- The OOC states represent isolated shifts of size $\{2.5 \text{ or } 3\} \times \text{sd}$, which are introduced to the IC sequences at one of the locations: $\{5, \text{ or } 15 \text{ or } 25\}$.

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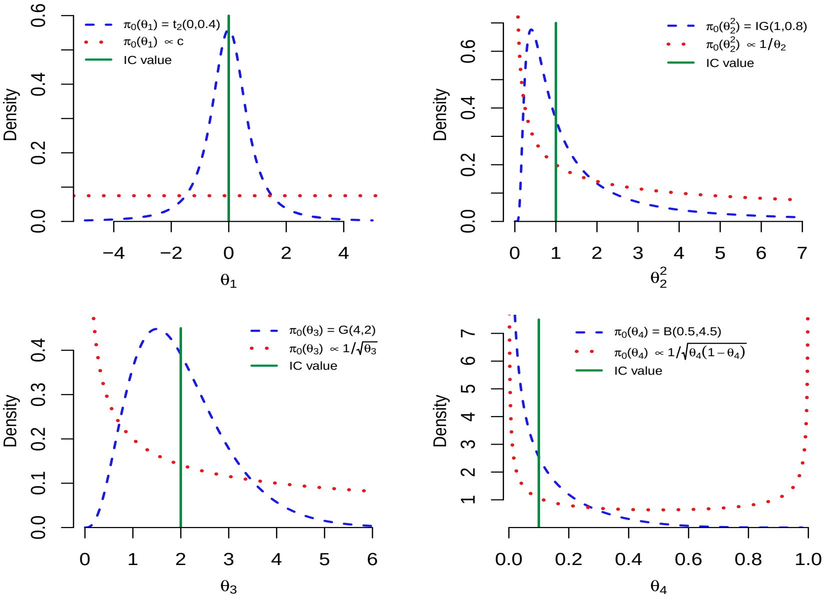
  - Binomial: reference prior $\pi_0\left(\theta_4\right) \propto 1/\sqrt{\theta_4(1-\theta_4)} \equiv Beta(1/2, 1/2)$ or the weakly informative $Beta(0.5, 4.5)$.

PCC: Initial Priors (Sensitivity Analysis)



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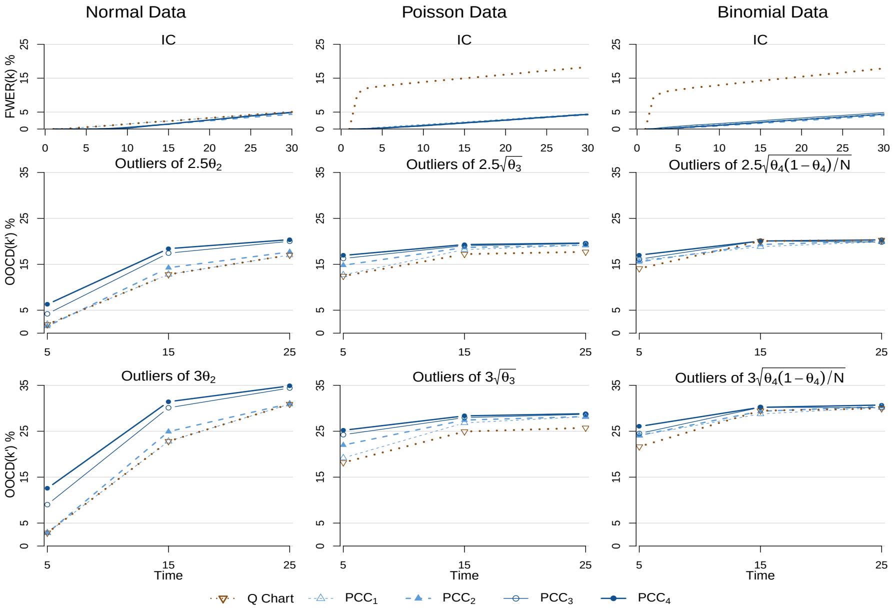

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- The goal is to accurately detect any transient parameter shift of large size, as this will have an impact on the reported patient results. Thus, it is of major importance to perform on-line monitoring of the process without a phase I exercise.
- We elicit the prior $\pi_0\left(\theta_1, {\theta_2}^2 | \boldsymbol{\tau}\right) \sim NIG\left(29.6, 1/7, 2, 0.56^2\right)$ and we had $n_0 = 30$ historical data (from a different reagent).

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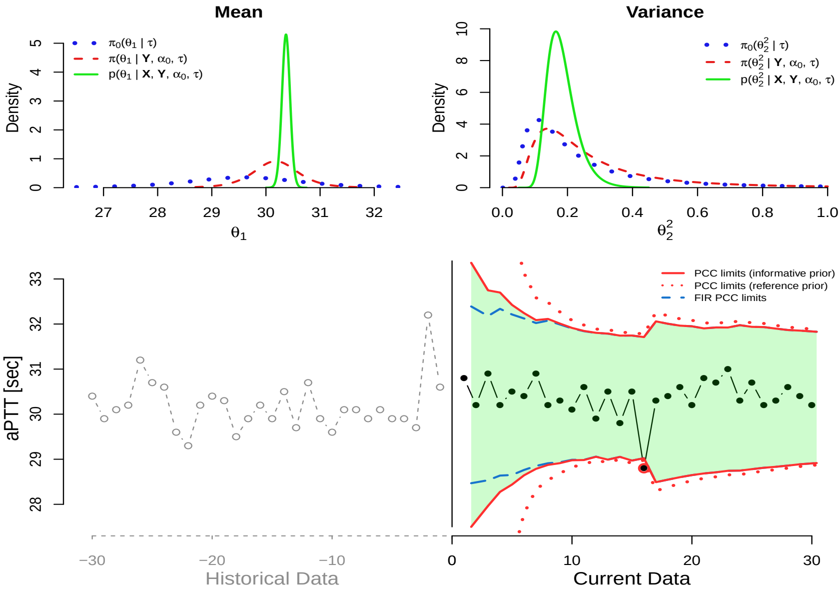

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- PCC illustration for discrete (Poisson) data. The data come from Hansen and Ghare (1987) and were also analyzed by Bayarri and García-Donato (2005).
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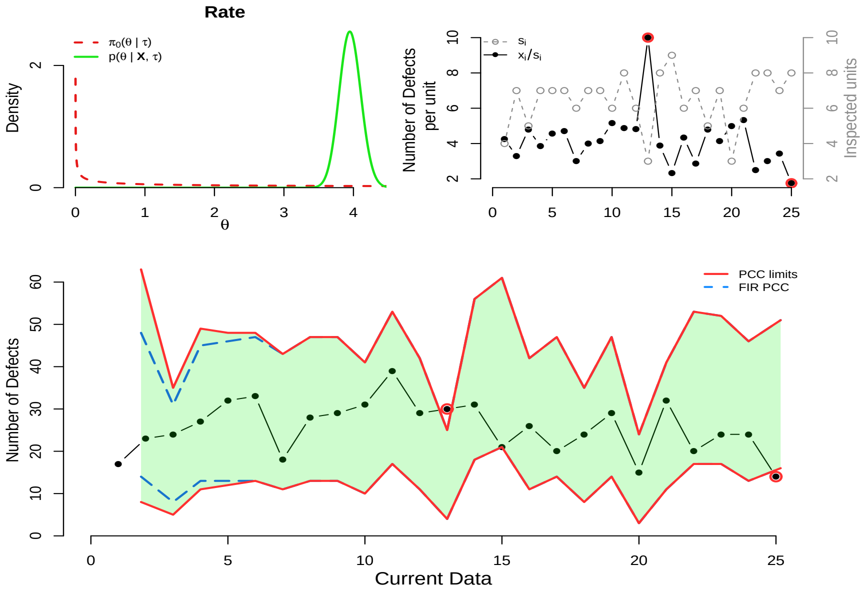
- We have 25 counts arriving sequentially that we will model using the Poisson distribution with unknown rate parameter, i.e. $X_i|\theta \sim P\left(\theta \cdot s_i\right)$.
- In this data set neither prior information regarding the unknown parameter nor historical data exist. Therefore, we use the reference prior as initial prior for θ, i.e. $\pi_0(\theta|\boldsymbol{\tau}) \propto 1/\sqrt{\theta} \equiv G(1/2, 0)$ and we also set $\alpha_0 = 0$ for the power prior term.

PCC Real Data Application (Industrial Setting)



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The END

Thank you!

Questions?