Non-parametric local capability indices for industrial planar artefacts

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Process Capability Indices (PCIs) are well known tools to estimate the mean-deviance performance of key product characteristics with respect to both targets and specification limits. The main aim of PCIs is to evaluate the instant quality of a process given the specification limits and they have become a fundamental tool also for commercial activity. Amongst different indicators of the process capacity, the Cpk (Montgomery, 2006) has gained popularity. Hereafter we considered the version of the Cpk proposed by Clements (1989) that generalises the usual specification of the index to account for potential asymmetry of the distribution of the target variable, indicated by Y from now on. In the case study that has motivated this paper, Y is the depth of the trench etched into a silicon wafer, a circular silicon support used in microelectronics to produce microchips. Clements' index is defined by the following equations:

$$C_{pu} = \frac{USL - \xi_{0.5}}{\xi_{0.99865} - \xi_{0.5}}, \qquad C_{pl} = \frac{\xi_{0.5} - LSL}{\xi_{0.5} - \xi_{0.00135}}, \qquad C_{pk} = \min(C_{pu}, C_{pl}), \qquad (1)$$

where ξ_τ is the quantile of order τ, $0 < \tau < 1$, of Y and LSL and USL represent the lower and upper specification limits of Y respectively. The C_{pk} index evaluates process capability at the aggregate level. In the application considered in this paper, for instance, a unique C_{pk} value is calculated for the entire wafer to assess the process ability to produce the output within the specification limits.

However, in planar artefacts, it can be relevant to assess process capability also at a local level i.e. at any spatial location of the manufact, in particular in those cases where the overall product surface has to be split in pieces to realise different items. This is, in fact, the case of microchip production since hundreds or even thousands of dice are obtained from each single wafer. In these circumstances deriving a local version of the C_{pk} can be worth of. Clearly the measurement effort to monitor the product process at this level is huge and a model-based approach can be adopted to estimate the index surface.

Let $\{Y(\boldsymbol{s}), \boldsymbol{s} \in D\}$ with $D \subseteq \mathbb{R}^2$ be a stochastic process indexed in the plane representing the characteristics of interest at any spatial location. In our case, D represents the silicon wafer i.e. a circle centred at the origin with a given radius and $Y(\boldsymbol{s})$ represented the depth of the trench at

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location $\boldsymbol{s}$. Indicating by $\xi_\tau(\boldsymbol{s})$ the quantile of order τ at location $\boldsymbol{s}$, a spatial version of the index in equation (1) is given by

$$C_{pu}(\boldsymbol{s}) = \frac{USL - \xi_{0.5}(\boldsymbol{s})}{\xi_{0.99865}(\boldsymbol{s}) - \xi_{0.5}(\boldsymbol{s})}, \quad C_{pl}(\boldsymbol{s}) = \frac{\xi_{0.5}(\boldsymbol{s}) - LSL}{\xi_{0.5}(\boldsymbol{s}) - \xi_{0.00135}(\boldsymbol{s})}, \quad C_{pk}(\boldsymbol{s}) = \min\left(C_{pu}(\boldsymbol{s}), C_{pl}(\boldsymbol{s})\right) \quad (2)$$

and can be straightforwardly estimated once one is able to provide an estimate of the relevant quantiles at each spatial location of interest.

Borgoni and Zappa (2020) provided such local estimates under the assumption that, conditionally to the spatial location, $Y(\boldsymbol{s})$ is log-normally distributed using a GAMLSS (Generalized Additive Models for Location Scale and Shape, Rigby et al. 2019) approach where all the parameters of the conditional distribution of the response variable are modelled as a function of the spatial coordinates of the point.

In this paper we propose an approach based on additive quantile models to estimate local C_{pk} values in a fully non-parametric manner i.e. without assuming any specific stochastic model for the target variable. We suggest modelling the quantiles of the depth of the trenches etched into a silicon wafer by a flexible function of spatial location $\boldsymbol{s}$. We adopt the approach suggested by Fasiolo et al. (2020) according to which a generalization of the Koenker loss function (1974) is used within a general Bayesian framework. The smooth relationship between regressors and the quantile of interest is handled using spline basis expansions.

Once the estimate $\hat{\xi}_\tau(\boldsymbol{s})$ is obtained for the appropriate quantiles at each desired $\boldsymbol{s}$, these values are plugged in equation (2) to obtain the C_{pk} estimate at that location.

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