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Connections between information measures and activation functions

In this work, we introduce a generalized measure of uncertainty, namely the cumulative information ψ -measure, in order to provide a unified perspective to the uncertainty framework. Indeed, it is a variability measure which reduces to several well-known information measures for appropriate choices of the function ψ . In particular, cumulative versions of Shannon and Tsallis entropies, Gini's dispersion indices, cumulative information generating functions, and additional new classes of uncertainty measures arise as special cases. We also investigate a suitable relative version, referred to as the relative cumulative information ψ -measure. We establish fundamental properties of the proposed measures, including monotonicity properties, bounds, and a set of related inequalities. We derive covariance-based and quantile-based representations, too. Illustrative examples are presented to clarify the behaviour of the measures under different model assumptions and choices of ψ . A notable feature of the proposed approach is the natural connection between the function ψ and certain increasing activation functions, commonly employed in neural networks. For instance, the function ψ associated with the cumulative paired Shannon entropy can be derived from the sigmoid activation function, and conversely. Motivated by this link, we introduce the generalized logistic-linear family of activation functions, which include classical models such as sigmoid and linear activations as particular cases within a parametric structure. Finally, we develop an application motivated by a neural network scenario to demonstrate the flexibility and novelty of the proposed framework. The results highlight the potential of the cumulative information ψ -measure as a bridge between information theory and modern neural networks.

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Primary authors: Prof. DI CRESCENZO, Antonio (Università degli Studi di Salerno); PISANO, Giulia (Università degli Studi di Salerno); Dr CAPALDO, Marco (RWTH Aachen University)

Presenter: PISANO, Giulia (Università degli Studi di Salerno)

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