Bayesian Hierarchical Modeling for Large-Scale Sensors Deployments and Applications

Anna Ballario[1,2], Francesca Pennecchi[1], Alessandro Schiavi[1], Andrea Prato[1]
[1]INRiM – Istituto Nazionale di Ricerca Metrologica
[2]PoliTo – Politecnico di Torino

ENBIS Spring Meeting 2026
Granada, May 27-28, 2026

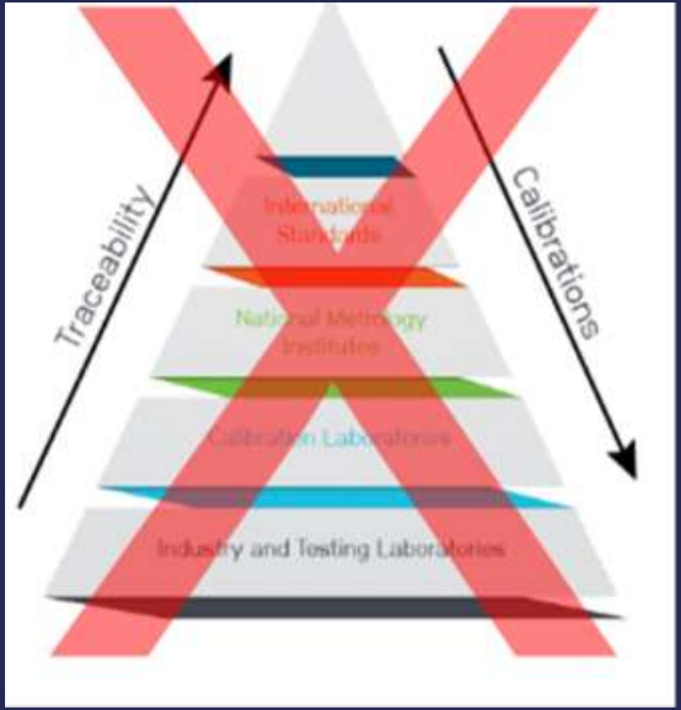
enbis European Network for Business and Industrial Statistics

INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

Politecnico di Torino

Introduction

- Millions of low-cost sensors being produced weekly:
  - → impossibility to apply standard calibration procedures
  - → large-scale calibration methods need to be introduced



- **Statistical methods**:
  - → in-lab calibration of a **small sample** and **virtual calibration** of the entire batch
  - → number of required experimental ("in-lab") calibrations reduced
  - → CONS: increased uncertainty and less reliability

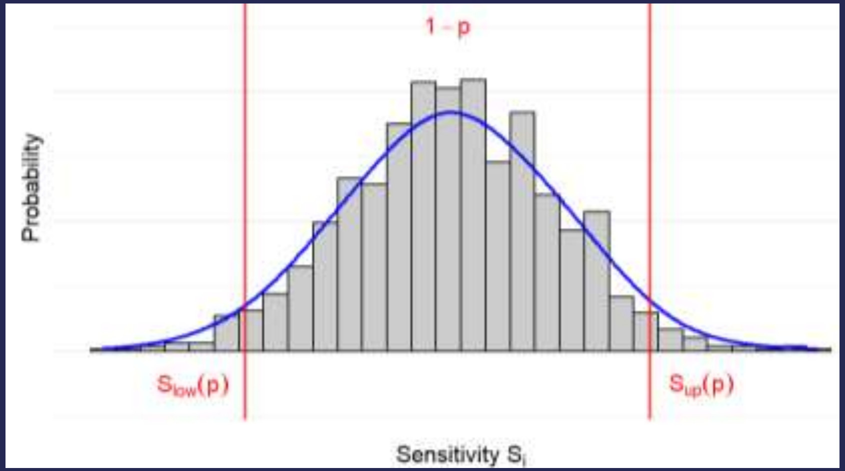
Virtual Calibration

- Exploiting data from a reference, fully in-lab calibrated lot of *N* sensors

  → mixture distribution of *N* Normal(S_i, $u(S_i)$) modeling the sensitivities of the reference lot

  → determination of an interval of acceptable sensitivity values [$S_{low}(p)$, $S_{up}(p)$]

- For any new lot from the same production, in-lab calibration of a small sample of sensors $n << N$
- Bayesian inference on the remaining *N-n* sensors
- Evaluation of the reliability and estimation of sensitivity



A Bayesian model

- Binomial prior to model the number of defective sensors *C* in the whole lot
- Hypergeometric likelihood on the number of defective items *k* in the sample of size *n*
- Conjugate model: Binomial posterior with updated parameters

$$f_{\text{post}}(C; k, n, N, p) = p^{C-k}(1-p)^{N-n-C+k}\binom{N-n}{C-k}$$

A metric for the reliability

A metric for batch reliability is defined based on posterior cumulative probability:

$$P_{\text{reliab,unk}} = \mathbb{P}_{\text{post}} = \sum_{C=k}^{C_{\text{bench}}} p^{C-k}(1-p)^{N-n-C+k}\binom{N-n}{C-k}$$

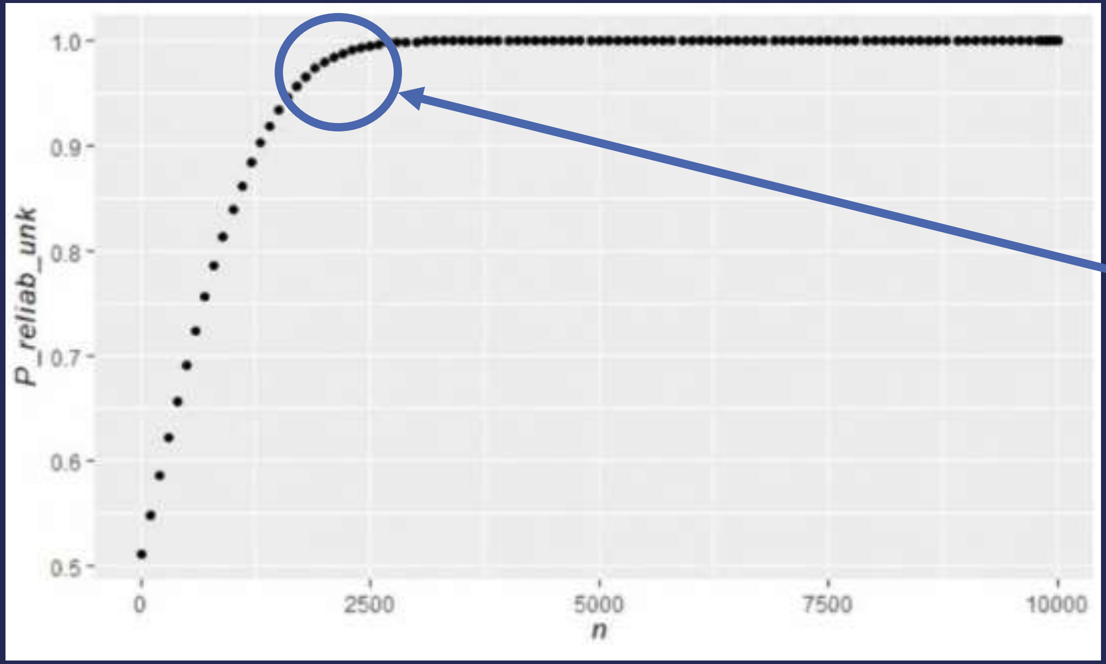


Figure. Variation of the probability $P_{reliab,unk}$ as a function of *n*

95 % of reliability with:

- at least *n* = 1630
- max *k* = 49

Hierarchical extension

- Introduction of a Beta hyperprior on the probability *p* of defective sensors

  ⟶ direct integration of prior industrial knowledge

  ⟶ explicit control over parameter variability

  ⟶ improvement in flexibility and robustness of the reliability

- CRUCIAL STEP: elicitation of hyperprior parameters to ensure the model coherently and accurately reflects prior knowledge

$$f_{\text{post}} = \binom{N-n}{C-k} \frac{\Gamma(C+\alpha)\,\Gamma(N-C+\beta)}{\Gamma(\alpha+\beta+N)} \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+k)\Gamma(n-k+\beta)}$$

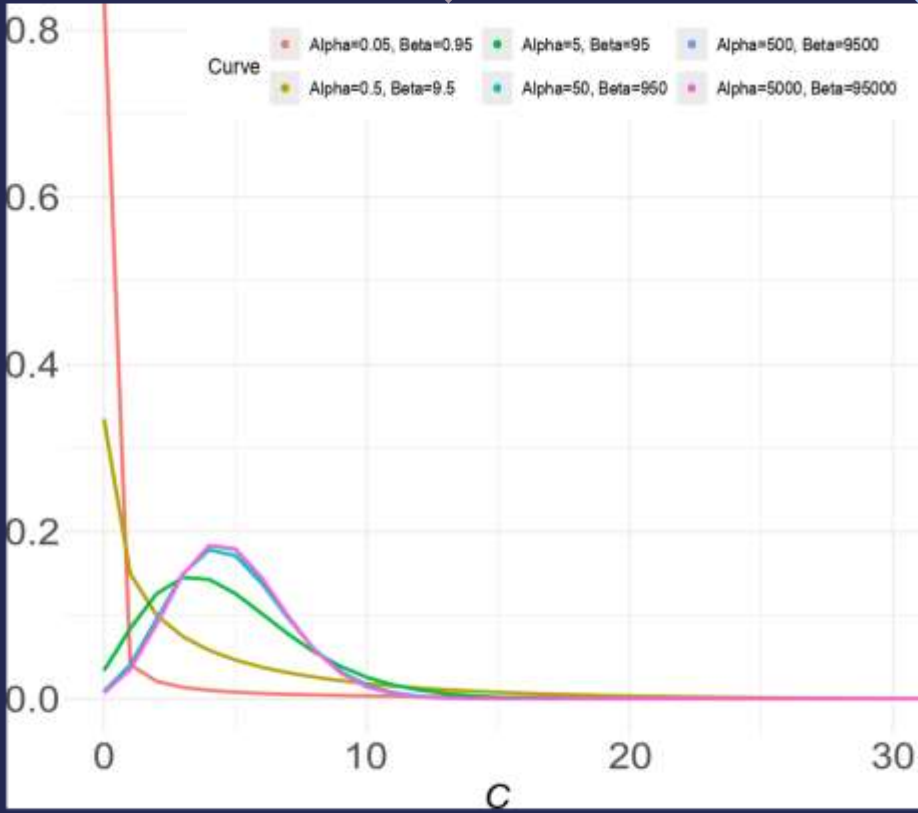
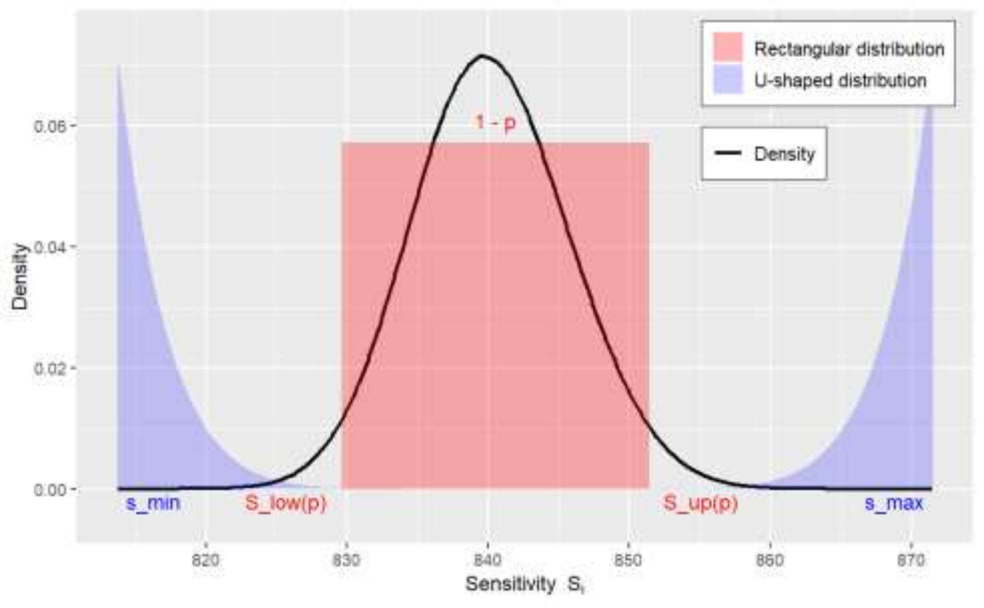


Figure. Comparsion of the posterior distribution for different Beta hyperprior

Sensitivity estimate and uncertainty evaluation

- Mixture distribution to model the sensitivity:
  - → $[S_{low}\ (p), S_{up}\ (p)]$: limits of an interval encompassing the $(1 - p)$ fraction of acceptable sensors
  - → $[S_{min}, S_{max}]$: maximum and minimum of the distribution
- S_{unk} is the variable to be estimated, arises from a mixture of:
  - → a rectangular distribution on $[S_{low}\ (p), S_{up}\ (p)]$
  - → a U-shaped distribution on $[S_{min}, S_{max}]$
- Weights of the mixture:-
  - $1 - \frac{C-k}{N-n}$: fraction of 'good' sensors
  - $\frac{C-k}{N-n}$: fraction of defective sensors



- Sensitivity:

$$S_{\mathrm{unk}}(p) = \frac{S_{\mathrm{low}}(p) + S_{\mathrm{up}}(p)}{2}$$

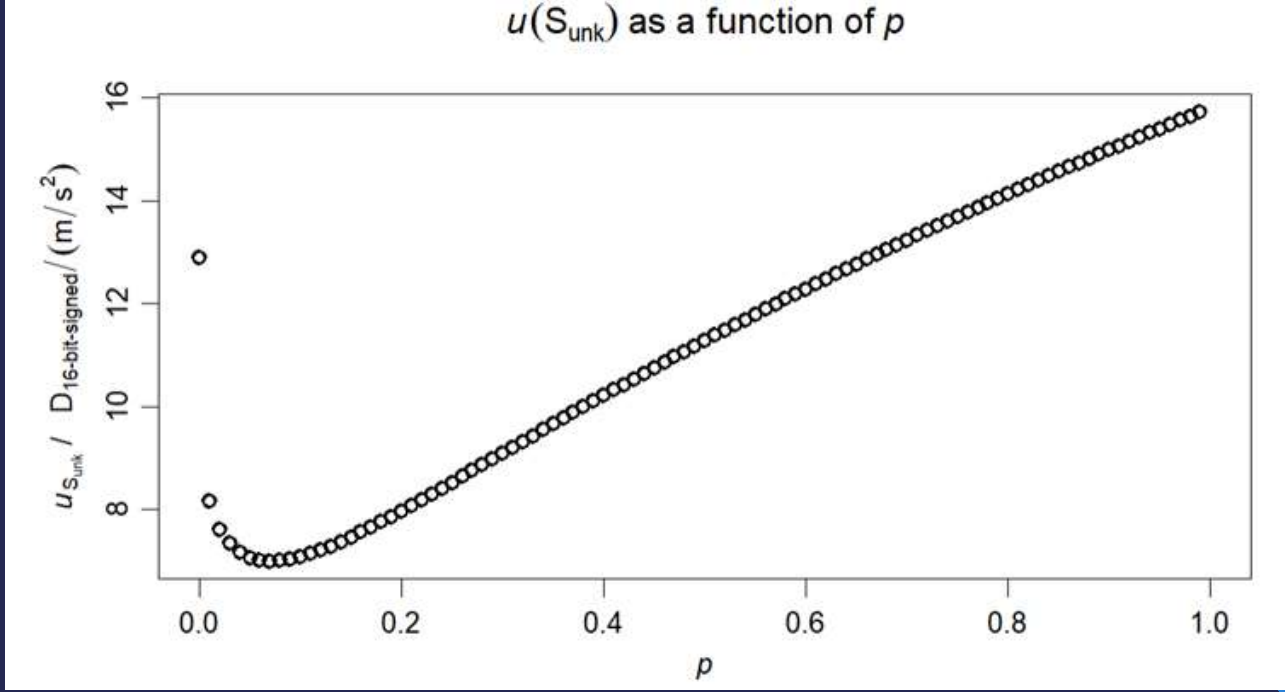


Figure. Value of the uncertainty as a function of *p*

- Uncertainty:

$$u(S_{\mathrm{unk}}) = \sqrt{\sum_{D=0}^{N-n} p_d \left[\left(1 - \frac{D}{N-n}\right) \frac{(S_{\mathrm{up}}(p) - S_{\mathrm{low}}(p))^2}{12} + \frac{D}{N-n} \frac{(S_{\max} - S_{\min})^2}{8} \right]}$$

where $p_d = \mathbb{P}(D = d)$

Case study

- 100 nominally identical MEMS calibrated at INRiM
- Dataset randomly split: 50 benchmark — 50 unknown batch
- Acceptance interval:

  $[S_{\mathrm{low}}, S_{\mathrm{up}}] = [833.18, 849.14]$ (for $p = 0.06$)
- Wider range for uncertainty evaluation:

  $[S_{\mathrm{min}}, S_{\mathrm{max}}] = [827.69, 855.06]$
- Estimate of the sensitivity: $S_{\mathrm{unk}} = 841.16\ \mathrm{D}_{\text{16-bit-signed}}/(\mathrm{ms}^{-2})$
- $n = 5$, $k = 0$ sensors counted outside acceptance interval
- True number of defective sensors: $C = 0$

Prior model	**Mean**	**Mode**	**Reliability**	$u(S_{\mathrm{unk}})$
Beta(0.06, 0.94)	0.4	0	0.959	4.684
Beta(0.6, 9.4)	1.8	0	0.821	4.910
Beta(6, 94)	2.6	2	0.730	5.035
Beta(60, 940)	2.7	2	0.717	5.054
Original Model	2.7	2	0.717	5.056

Conclusion

- Industrial impact: drastically reduced times and costs for sensors network deployment while maintaining traceability

- Hierarchical modeling insight: better captures prior information structure, but highly sensitive to hyperparameter choice; weakly informative priors may lead to high reliability while increasing posterior variance

- A decision framework employing utility functions is under development to provide optimal sampling plans while keeping into account posterior variance

References

[1] A. Prato, F. Pennecchi, G. Genta and A. Schiavi, A Bayesian statistical method for large-scale MEMS-based sensors calibration, Metrologia 61 (2024) 015005

[2] Prato A, Mazzoleni F and Schiavi A, Traceability of digital 3-axis MEMS accelerometer: simultaneous determination of main and transverse sensitivities in the frequency domain, Metrologia 57 (2020) 035013

[3] Ballario A., Prato A., Schiavi A., Pennecchi F., Bayesian Hierarchical Modelling for Large-Scale Sensor Calibration, submitted to Quality and Reliability Engineering International, under review

Thank you for the attention!

INRiM
ISTITUTO NAZIONALE
DI RICERCA METROLOGICA

Politecnico
di Torino