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Random Designs for Quantisation in High Dimension

Incremental design for computer experiments traditionally relies on space-filling or uniformity criteria, but these become impractical in high dimensions. The greedy minimisation of the L_s-mean quantisation error (or distortion) offers a valuable alternative, though it is also computationally intractable for large d.

This talk focuses on random designs composed of i.i.d. points sampled from a specific distribution. For large d, observing the behaviour predicted by Zador's theorem requires an impractically large sample size n, growing super-exponentially with d. We address this challenge by analysing the quantisation problem for spherically symmetric distributions. Our results show that for moderate n random quantisers uniformly distributed on a sphere of suitable radius R achieve exceptional performance. The expected distortion, computed exactly via a triple integral, allows numerical optimisation of R. Using extreme-value theory, we also derive approximations for R, revealing that, when n grows with d and $d \to \infty$, R may converge to zero or approach a limiting value R_∞ independent of s, depending on the growth rate of n.

While spherically symmetric distributions may seem restrictive, they provide a starting point for further applications, such as quantising the uniform measure on the hypercube $X = [-1, 1]^d$. For large d, the uniform measure on X can be approximated by a spherically symmetric distribution, and one can consider random quantisers distributed according to a product measure, which can itself be approximated by a spherically symmetric distribution. Preliminary results suggest that quantisers distributed on the vertices of a smaller hypercube exhibit promising performance.

Special/ Invited session

Classification

Mainly methodology

Keywords

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