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Time series models for cultural heritage preservation: The case of the Brunelleschi's Dome

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Introduction

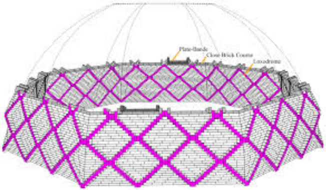
Background

Santa Maria del Fiore is the cathedral of Florence, begun in 1296 in the Gothic style to the design of Arnolfo di Cambio and completed in 1436 with the **dome engineered by Filippo Brunelleschi**

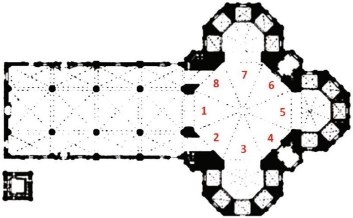
Brunelleschi's Dome

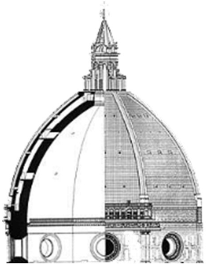
Brunelleschi's Dome has a particular structure consisting of two domes (**an inner dome and an outer dome**) that are **self-standing** in virtue of a particular layout of the bricks → **"fishbone layout of bricks"**

Fishbone layout of bricks

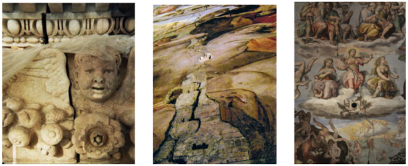


Plan section and inner and outer dome





Web cracks in the Dome



- **Firsts cracks** in the dome appeared **at the end of the 15th century**
- **Cracks are now present in all the 8 webs**

  → cracks are mainly present in the webs 4 and 6, both at the opposite of the nave

  → webs 1, 3, 5, and 7 show fewer cracks, perhaps because they are protected by the nave (web 1) or by the side chapels surrounding the church (webs 3, 5 and 7)

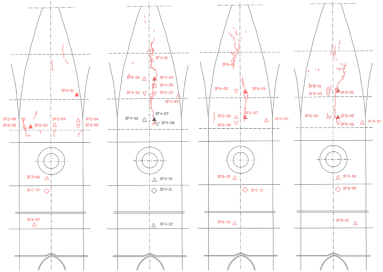
The monitoring system

- Brunelleschi's Dome is the only large dome of the Renaissance that had not yet been protected by actions of containment (rigid structures), however, it is closely monitored using a complex system of sensors
- Currently, the main part of the instruments installed inside the Dome are **deformometers** positioned upon the cracks to measure their width

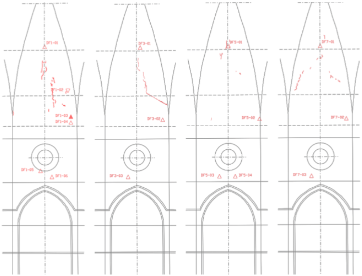
- Two main monitoring systems based on deformometers
  - a detection system based on 22 mechanical devices, installed in 1955
    $\rightarrow$ 4 measures per year
    $\rightarrow$ 93 data per deformometer, for a total of $4,356$ measurements
  - a detection system based on 66 electronic sensors, installed **in 1988**
    $\rightarrow$ 4 measures per day
    $\rightarrow$ $35,100$ data per deformometers, for **a total of about 6 million measurements**
  - Positioning of deformometers on the Dome is characterized by: the web (1 to 8), the dome (inner or outer), the horizontal sector of the Dome (1 to 5), and the intrados or extrados
- Other instruments installed inside the Dome measure the **masonry temperature**, the **air temperature**, the **humidity**, and (but only since a few years) the seismic tremors

Location of the main cracks and instruments

Even webs



Odd webs



Aim of the contribution

- Due to the large dimensionality of the data, previous studies on the Dome either restricted to a few cracks (often, just one crack) and/or to a single year
- Moreover, previous studies addressed environmental factors only to a limited extent

Here, we aim at

- Explaining the relationships between individual cracks and environmental variables
- Investigating the interconnections among neighbouring cracks

Statistical time-series models applied to multiple sensor data

Statistical methodology

Data

- 58 electronic deformometers and 47 masonry thermometers in the period October 2009 - October 2017 (about 2600 daily obs.)
- Daily records of air temperature and humidity from “Il Meteo” website
- Earthquake data sourced from the website of the Italian National Institute of Geophysics and Volcanology (INGV) Strong Motion Database

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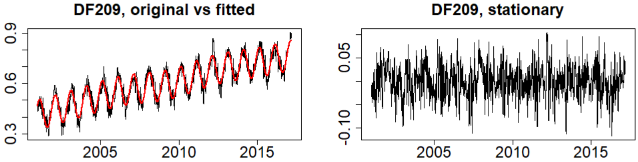
- Treatment of missing values and outliers through a cascade imputation procedure involving linear regression models and quadratic sinusoidal regressions (to account for long-term seasonal fluctuations)
- De-trended and de-seasonalized time series



Relationship between cracks and environmental variables

To examine the relationship between crack width and environmental variables (humidity, temperature, and seismic activity) we estimate a **univariate ARIMA model for each deformometer**

$$\mathrm{w}_t = \mathbf{X}_t'\beta + \frac{\theta_q(L)}{\phi_p(L)}\varepsilon_t$$

- w_t: de-trended and de-seasonalized observation of a single deformometer
- $\phi_p(L)$ and $\theta_q(L)$: lag polynomials having all roots outside the unit circle
- $\mathbf{X}_t$: $(K \times 1)$ vector of explanatory variables
- β: $(K \times 1)$ vector of coefficients
- ε_t: white noise with mean 0 and variance σ_ε^2
- p and q are selected by minimising BIC

Interconnections among cracks

To capture interdependencies among neighbouring deformometers, we fit **Structural VAR models with covariates (SVARX) to small clusters of deformometers**

$$\mathbf{A}\Phi_p(L)\mathbf{W}_t = \mathbf{A}\mathbf{B}_j(L)\mathbf{X}_t + \mathbf{A}\varepsilon_t$$

- $\mathbf{W}_t$: de-seasonalized and de-trended web crack measurements of deformometers belonging to the same cluster
- $\Phi_p(L)$ and $\mathbf{B}_j(L)$: matrices polynomial in the lag operator L
- $\Phi_1, \dots, \Phi_p$ and $\mathbf{B}_0, \dots, \mathbf{B}_j$: $N \times N$ and $N \times K$ coefficient matrices
- $\mathbf{X}_t$: K-dimensional vector of explanatory variables
- $\varepsilon_t \sim N(0, \Sigma)$: multivariate white noise
- $\mathbf{A}$: $N \times N$ lower triangular matrix modelling the instantaneous dependencies between the masonry temperature variation and the deformometers

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	DF408	DF407	DF406	
$\mathbf{A} =$	1	0	0	DF408
	a_{21}	1	0	DF407
	a_{31}	a_{32}	1	DF406

Based on the estimated SVARX models, we compute

- **Impulse Response Functions (IRF)**: to describe the response over time of an endogenous variable included in the system (e.g., a certain DF) to an isolated impulse from another endogenous variable in the same system (another DF)
- **Transfer Functions (TF)**: to describe the response over time of endogenous variables to an external impulse originating from the exogenous variables
  $\rightarrow$ Our main interest lies in the effect of shocks on masonry temperature

As a robustness check, we also estimate the **Generalized Impulse Response Functions (GIRFs)**, tracing shock responses without imposing an ordering of variables.

Results

Effect of exogenous variables - I

- An increase in **humidity** is significantly associated with a reduction of the amplitude of the web cracks
- An increase in the within-day **air temperature variation** is significantly associated with an expansion of the crack width
- The strength of earthquakes does not seem to have a significant impact on the web crack evolution

Effect of exogenous variables - II

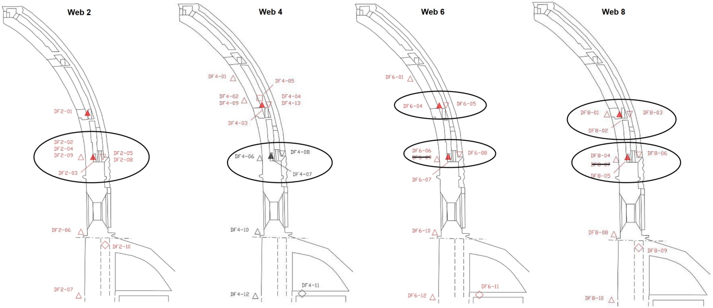
As for the effect of the **masonry temperature**, we need to distinguish between even and odd webs

- In odd webs, all the deformometers placed on the inner dome tend to widen as the masonry temperature increases, whereas all the deformometers located in the tambour tend to shrink when the masonry temperature increases; deformometers on the outer dome are absent
- In even webs, as the masonry temperature increases, the deformometers on the outer dome tend to shrink while those on the inner dome tend to expand; moreover - as opposed to the odd webs - deformometers located in the tambour tend to widen for positive variations in the masonry temperature

The higher number of instruments located on even webs permits further considerations $\longrightarrow$ **structural relations among cracks**

Structural relations among cracks

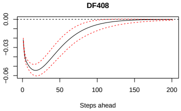
Focus on the **clusters of deformometers** emerging from the ARIMA results

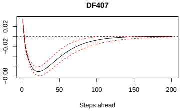


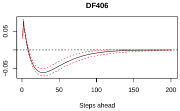
- Web 2: DF202, DF203 and DF208; DF204 and DF205
- Web 4: DF406, DF407 and DF408; DF402, DF403, DF404 and DF413
- Web 6: DF606, DF607 and DF608; DF604 and DF605
- Web 8: DF804, DF805 and DF806; DF801, DF802 and DF803

TF and IRF for web 4

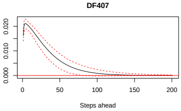
TF from $\Delta MT4$

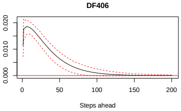




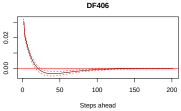


IRF from DF408



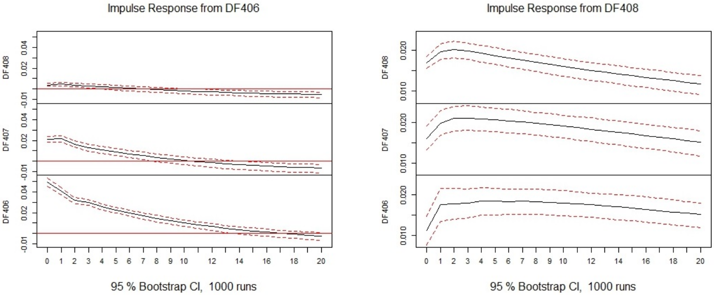


IRF from DF407



GIRF for web 4

If our structural assumption regarding the directionality of shocks is valid, we would expect the GIRFs from internal (e.g. DF406) to external (e.g. DF408) cracks to be statistically insignificant.



⟶ GIRFs align with IRFs in terms direction and magnitude of effects.

- When masonry temperature increases, the **external cracks** in webs 4, 6, and 8 shrink
- The **internal cracks** are influenced directly by variations in masonry temperature as well as by neighbouring external cracks
  - a positive thermal shock induces a pronounced initial expansion followed by a shrinkage
  - a positive shock in the external cracks of webs 4, 6 and 8 induces the internal ones to expand accordingly
- In the long run (100 days), the responses to both temperature shocks and shocks from neighbouring cracks tend to vanish
- Cracks on web 2 tend to have opposite reactions

Conclusions

Main conclusions

- This work represents a first effort to apply statistical methods for time series analysis to the field of **static Structural Health Monitoring (SHM)** for the analysis of data coming from **multiple sensors** positioned on historical heritage buildings
- Our findings enhance the **understanding of the Dome's structural stability**, identifying distinct behaviors between even and odd webs and highlighting differences between the inner and outer dome in terms of reaction to thermal shocks
- The monitoring approach proposed in this study can be readily adapted and replicated on other monuments, offering a **scalable framework** to safeguard cultural heritage through rigorous statistical methodologies

Role of statistical methods for the preservation of historical heritage buildings

Limits and ongoing research

The present modeling approach ignores the information about the **spatial position** of the deformemeters

- VAR models estimated on the whole Dome with a penalised version of the Adaptive Lasso estimator, where in the penalisation part of the model, we introduce a matrix of weights to take into account the spatial positions of the deformometers
- State-space models
- Models for circular data

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Thank you for your attention!

For any question, please contact me at:

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